Using Mixed Effects Models in Psychology

Scott Fraundorf
sfraundo@pitt.edu

Office: 608 LRDC
Office hours: Tu 3-4, Wed 1-2, or by appointment
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
    - Nested random effects
    - Crossed random effects
  - Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
My introduction

- Psychology, cognitive area, 3\textsuperscript{rd} year at Pitt
  - Office here in Learning Research and Development Center

- Research interests:
  - Language processing
  - Memory & metacognition

- Enjoy teaching stats!
Course Goals

- We will:
  - Understand form of mixed effects models
  - Apply mixed effects models to common designs in psychology (e.g., factorial experiments, educational interventions, longitudinal studies)
  - Fit mixed effects models in R using lme4
  - Diagnose and address common issues in using mixed effects models

- We won’t:
  - Cover algorithms used by software to compute mixed effects models
Course Requirements

- Midterm project:
  - Analyze a paper in your research area that uses mixed effects models
  - We will have a class discussion on current standards for models & reporting

- Final project:
  - Analyze a dataset of your own & report what you did
  - In-class presentation

- Weekly readings
  - Available on CourseWeb
Course Requirements

- We’ll be fitting models in R
  - Free & runs on basically any computer
  - Next week, will cover basics of using R
Mixed Effects Models Intro

- Course goals & requirements
  - Motivation for mixed effects models
    - Multiple random effects
      - Nested random effects
      - Crossed random effects
    - Categorical data
    - Continuous predictors
  - Big picture view of mixed effects models
  - Terminology
  - R
Why Mixed Effects Models?
Mixed Effects Models Intro

- Course goals & requirements
  - Motivation for mixed effects models
    - Multiple random effects
      - Nested random effects
      - Crossed random effects
    - Categorical data
    - Continuous predictors
  - Big picture view of mixed effects models
- Terminology
- R
Problem 1: Multiple Random Effects

- Inferential statistics you may be familiar with:
  - ANOVA
  - Regression
  - Correlation
- All of these methods involve random sampling out of a larger population
- To which we hope to generalize

Subject 1  
Subject 2  
Subject 3
Inferential statistics you may be familiar with:
- ANOVA
- Regression
- Correlation

Standard assumption: All observations are independent

Subject 1’s score doesn’t tell us anything about Subject 2’s

Problem 1: Multiple Random Effects
Important!

- Impressive if the 20 people who did a practice test learned better than the 20 people who reread the textbook.
- *Not* so impressive if we learn those 20 people compared notes outside of the experiment.
- They will all do well or do poorly.

Problem 1: Multiple Random Effects
**Problem 1: Multiple Random Effects**

- Important!
  - Also *not* so impressive if the 20 Test subjects were all in the same biology section and the 20 Restudy subjects were in a different section
- Need to account for differences in instructor, time of day
Problem 1: Multiple Random Effects

- Independence assumption is fair if we randomly sample 1 person at a time
  - e.g., you recruit 40 undergrads from the Psychology Subject Pool
- But maybe this isn’t all we should be doing… (Henrich et al., 2010, Nature)
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
    - Nested random effects
    - Crossed random effects
  - Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
But many sensible, informative research designs involve more complex sampling procedures

Example: Sampling multiple children from the same family
  - Kids from the same family will be more similar
  - a/k/a clustering

Problem 1A: Nested Random Effects
Problem 1A: Nested Random Effects

- But many sensible, informative research designs involve more complex sampling procedures
- Or: Kids in classrooms in schools
  - Kids from the same school will be more similar
  - Kids in same classroom will be even more similar!
One way to describe what’s going on here is that there are several levels of sampling, each **nested** inside each other.

- **SAMPLED SCHOOLS**
  - **SAMPLED CLASSROOMS** in those schools
  - **SAMPLED STUDENTS** in those classrooms
- Each level is what we’ll call a **random effect** (a thing we sampled)
Two challenges:
- Statistically, we need to take account for this non-independence (similarity)
- Even a small amount of non-independence can lead to spurious findings (Quené & van den Bergh, 2008)
- We might want to characterize differences at each level!
- Are classroom differences or school differences bigger?
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
  - Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
Problem 1B: Crossed Random Effects

- A closely related problem shows up in many experimental studies
- Experimental / research materials are often sampled out of population of possible items
  - Words or sentences
  - Educational materials
  - Hypothetical scenarios
  - Survey items
  - Faces
Problem 1B: Crossed Random Effects

• We might ask:
  • Do differences in stimuli used account for group / condition differences?
  • e.g., Maybe easier vocab words used in one condition

Maintenance rehearsal

Elaborative rehearsal

2. Wikipedia is usually my first resource for research.

strongly agree
agree
neutral
disagree
strongly disagree
Problem 1B: Crossed Random Effects

- We might ask:
  - Do differences in stimuli used account for group / condition differences?
  - Do our results generalize to the population of all relevant items?
    - All Spanish vocab words
    - All fictional resumes
    - All questionnaire items that measure extraversion
    - All faces
Problem 1B: Crossed Random Effects

- Again, we are sampling two things—subjects and items
- Arrangement is slightly different because each subject gets each item
  - Crossed random effects
- Still, problem is that we have multiple random effects (things being sampled)
Problem 1B: Crossed Random Effects

- Robustness across stimuli has been a major concern in psycholinguistics for a long time

The Language-as-Fixed-Effect Fallacy: A Critique of Language Statistics in Psychological Research

Herbert H. Clark
Stanford University

- Now growing interest in other fields, too

Treating Stimuli as a Random Factor in Social Psychology: A New and Comprehensive Solution to a Pervasive but Largely Ignored Problem

Charles M. Judd and Jacob Westfall
University of Colorado Boulder

David A. Kenny
University of Connecticut
Problem 1B: Crossed Random Effects

- OLD ANOVA solution: Do 2 analyses
  - Subjects analysis: Compare each subject (averaging over all of the items)
    - Does the effect generalize across subjects?
  - Items analysis: Compare each item (averaging over all of the subjects)
    - Does the effect generalize across items?

Note: not real data

### SUBJECT ANALYSIS
\[ F_1(1,3) = 18.31, \ p < .05 \]

<table>
<thead>
<tr>
<th>Subject</th>
<th>Control</th>
<th>Primed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scott</td>
<td>1305</td>
<td>1122</td>
</tr>
<tr>
<td>Tessa</td>
<td>1100</td>
<td>905</td>
</tr>
<tr>
<td>Natasha</td>
<td>950</td>
<td>900</td>
</tr>
<tr>
<td>Chuck</td>
<td>1070</td>
<td>988</td>
</tr>
</tbody>
</table>

### ITEM ANALYSIS
\[ F_2(1,4) = 22.45, \ p < .01 \]

<table>
<thead>
<tr>
<th>Item</th>
<th>Control</th>
<th>Primed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knight</td>
<td>1100</td>
<td>883</td>
</tr>
<tr>
<td>Monkey</td>
<td>979</td>
<td>930</td>
</tr>
<tr>
<td>Bird</td>
<td>1114</td>
<td>912</td>
</tr>
<tr>
<td>Vacation</td>
<td>1094</td>
<td>830</td>
</tr>
<tr>
<td>Pirate</td>
<td>1251</td>
<td>939</td>
</tr>
</tbody>
</table>
**Problem 1B: Crossed Random Effects**

- **OLD ANOVA solution:**
  - Do 2 analyses
    - *Subjects analysis*
    - *Items analysis*

- **Problem:** We now have 2 different sets of results. Might conflict!
  - Possible to combine them with \( \min F' \), but not widely used

---

<table>
<thead>
<tr>
<th>Subject</th>
<th>Control</th>
<th>Primed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scott</td>
<td>1305</td>
<td>1122</td>
</tr>
<tr>
<td>Tessa</td>
<td>1100</td>
<td>905</td>
</tr>
<tr>
<td>Natasha</td>
<td>950</td>
<td>900</td>
</tr>
<tr>
<td>Chuck</td>
<td>1070</td>
<td>988</td>
</tr>
</tbody>
</table>

**SUBJECT ANALYSIS**
\[
F_1(1,3) = 18.31, \ p < .05
\]

<table>
<thead>
<tr>
<th>Item</th>
<th>Control</th>
<th>Primed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knight</td>
<td>1100</td>
<td>883</td>
</tr>
<tr>
<td>Monkey</td>
<td>979</td>
<td>930</td>
</tr>
<tr>
<td>Bird</td>
<td>1114</td>
<td>912</td>
</tr>
<tr>
<td>Vacation</td>
<td>1094</td>
<td>830</td>
</tr>
<tr>
<td>Pirate</td>
<td>1251</td>
<td>939</td>
</tr>
</tbody>
</table>

**ITEM ANALYSIS**
\[
F_2(1,4) = 22.45, \ p < .01
\]
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
  - Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
Problem 2: Categorical Data

- ANOVA assumes our response is *continuous*

  RT: 833 ms

- But, we often want to look at *categorical* data

  - do you like me?
    - yes
    - no

  - Does student graduate high school or not?
  - Item recalled or not
  - Region fixated in eye-tracking experiment
Problem One: Categorical Data

- Traditional solution:
  - Analyze proportions
    - Maybe with some transformation (e.g., arcsine, logit)
  - Violates assumptions of ANOVA
    - Among other issues: ANOVA assumes normal distribution, which has infinite tails
    - But proportions are clearly bounded
    - Model could predict impossible values like 110%

Problem 2: Categorical Data

<table>
<thead>
<tr>
<th>Maintenance rehearsal</th>
<th>0.43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elaborative rehearsal</td>
<td>0.65</td>
</tr>
</tbody>
</table>

But

0 ≤ proportions ≤ 1
Problem 2: Categorical Data

- Traditional solution: Analyze proportions
- Model could predict impossible values like 110%

<table>
<thead>
<tr>
<th>Maintenance rehearsal</th>
<th>0.43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elaborative rehearsal</td>
<td>0.65</td>
</tr>
</tbody>
</table>

But 0 ≤ proportions ≤ 1
Problem 1: Categorical Data

Traditional solution: Analyze proportions

Violates assumptions of ANOVA

Can lead to:
- Spurious effects (Type I error)
- Missing a true effect (Type II error)

<table>
<thead>
<tr>
<th>Maintenance rehearsal</th>
<th>Elaborative rehearsal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
- Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
Many interesting independent variables vary continuously:
- e.g., Word frequency
Problem 3: Continuous Predictors

- Many interesting independent variables vary continuously
  - Or: Second language proficiency or reading skill

- ANOVAs require division into categories
  - e.g., median split
Many interesting independent variables vary continuously
- Or: Second language proficiency or reading skill

ANOVAAs require division into categories
- e.g., median split
- Or: extreme groups design

**Problem 3: Continuous Predictors**
Many interesting independent variables vary continuously

- Or: Second language proficiency or reading skill

ANOVA\textsuperscript{s} require division into categories

- Problem: Can only ask “is there a difference?”, not form of relationship
- Loss of statistical power (Cohen, 1983)
The Berenstain Bears

AND THE TROUBLE WITH ANOVA

Stan & Jan Berenstain
Mixed Effects Models

- Course goals & requirements
- Motivation for mixed effects
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
- Categorical data
- Continuous predictors
- Big picture view of mixed effects
- Terminology
- R
Mixed Effects Models to the Rescue!

- Biggest contribution of mixed-effects models is to incorporate *multiple random effects* into the *same analysis*.

How does the effect of parental stress on screen time generalize across *children* and *families*?

How does the effect of aphasia on sentence processing generalize across *subjects* and *sentences*?
Mixed Models to the Rescue!

- Model outcome using regression-like approach

\[
\text{GPA} = \text{Motivation} + \text{Subject} + \text{School}
\]

Outcome

Problem 1A solved!
Mixed Effects Models to the Rescue!

- For many experimental designs, this means a change in what gets analyzed
  - ANOVA: Unit of analysis is *cell mean*

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>PRIMED</th>
<th>CONTROL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scott</td>
<td>960</td>
<td>1214</td>
</tr>
<tr>
<td>Ben</td>
<td>913</td>
<td>1003</td>
</tr>
</tbody>
</table>

- **Mixed effects models**: Unit of analysis is *individual trial!*

<table>
<thead>
<tr>
<th>RT</th>
<th>PRIME?</th>
<th>SUBJECT</th>
<th>ITEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1224</td>
<td>Yes</td>
<td>Scott</td>
<td>eagle</td>
</tr>
<tr>
<td>683</td>
<td>No</td>
<td>Scott</td>
<td>penguin</td>
</tr>
<tr>
<td>432</td>
<td>Yes</td>
<td>Scott</td>
<td>pitohui</td>
</tr>
<tr>
<td>892</td>
<td>No</td>
<td>Scott</td>
<td>robin</td>
</tr>
<tr>
<td>1028</td>
<td>Yes</td>
<td>Ben</td>
<td>penguin</td>
</tr>
</tbody>
</table>
Mixed Models to the Rescue!

- Model outcome using regression-like approach
- Look at *individual trials/observations* (not means)

Semantic categorization: Is it a bird?

Problem 1B solved!
Mixed Models to the Rescue!

- In a regression, easy to include independent variables that are continuous

\[ \text{RT} = \text{Lexical Freq.} + \text{Subject} + \text{Item} + 2.32 \]

Semantic categorization: Is it a bird?

Problem 3 solved!
Mixed Models to the Rescue!

- Link functions allow us to relate model to DV that *isn’t* normally distributed

Accuracy

Odds of correct response on this trial (yes or no?)

= Lexical Freq. + Subject + Item

Problem 2 solved!
Mixed Models to the Rescue!

- Link functions allow us to relate model to DV that *isn’t* normally distributed

Yes or No?

Odds of graduating college

Motivation

Motivational intervention

+ Subject

+ School

Problem 2 solved!
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
- Categorical data
- Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
A Terminological Note…

- “Mixed effects models” is not the most precise term
  - Technically, any model that includes subjects, classrooms, or items (a “random effect”) plus experimental variables (“fixed effects”)

- Models we’ll be talking about are hierarchical linear models

- But “mixed effects models” has caught on in cognitive psychology
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
- Categorical data
- Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
How do we run mixed effects models?

- Multiple software packages could be used to fit the same conceptual model
- Most popular solution: R with *lme4*
**R Pros**

- Free!
- Runs on any computer
- Lots of add-ons—can do just about any type of model
- Gaining popularity
- Makes analyses clear & more reproducible

**R Cons**

- Documentation / help files not the best
  - Other online resources
- Requires some programming, not just menus
Two Ways to Use R

- Regular R  < www.r-project.org >
- RStudio  < www.r-studio.com >
  - Different interface
    - Some additional windows/tools to help you keep track of what you’re doing
  - Same commands, same results
  - Also available for just about any platform
  - Recommended (but not required)
  - Requires you to download regular R first
- Also R Commander with buttons, menus
  - No experience with this, not sure if it works with lme4
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
  - Nested random effects
  - Crossed random effects
- Categorical data
- Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
Wrap-Up

- Mixed effects models solve three common problems with ANOVAs
  - Multiple random effects (subjects, items, classrooms, schools)
  - Categorical outcomes
  - Continuous predictors
- For next week: Download R!
  - Next class, we’ll get started using R
  - If sitting in, e-mail me (sfraundo@pitt.edu) for CourseWeb access
- CourseWeb survey about your research & statistical background