Using Mixed Effects Models in Psychology

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Office: 608 LRDC
Office hours: Tu 12:30-1:30, Wed 1-1:30, or by appointment
Mixed Effects Models Intro

- Course goals & requirements
- Motivation for mixed effects models
  - Multiple random effects
    - Nested random effects
    - Crossed random effects
  - Categorical data
  - Continuous predictors
- Big picture view of mixed effects models
- Terminology
- R
My introduction

- Psychology, cognitive area, 5th year at Pitt
  - Office here in Learning Research and Development Center

- Research interests:
  - Memory & metacognition
  - Language processing

- Enjoy teaching stats!
**Course Goals**

- **We will:**
  - Understand form of mixed effects models
  - Apply mixed effects models to common designs in psychology and related fields (e.g., factorial experiments, educational interventions, longitudinal studies)
  - Fit mixed effects models in R using lme4
  - Diagnose and address common issues in using mixed effects models

- **We won’t:**
  - Cover algorithms used by software to compute mixed effects models
Course Requirements

- **Midterm project:**
  - Analyze a paper in your research area that uses mixed effects models
  - We will have a class discussion on current standards for models & reporting
- **Final project:**
  - Analyze a dataset of your own & report what you did
  - In-class presentation
- **Weekly readings**
  - Available on CourseWeb
Course Requirements

- We’ll be fitting models in R
  - Free & runs on basically any computer
  - Next week, will cover basics of using R
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Why Mixed Effects Models?
Mixed Effects Models Intro

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Inferential statistics you may be familiar with:

- ANOVA
- Regression
- Correlation

All of these methods involve random sampling out of a larger population.

To which we hope to generalize.

**Problem 1: Multiple Random Effects**
• Inferential statistics you may be familiar with:
  • ANOVA
  • Regression
  • Correlation

• Standard assumption: All observations are independent

• Subject 1’s score doesn’t tell us anything about Subject 2’s
Problem 1: Multiple Random Effects

- Important!
  - Impressive if the 20 people who did a practice test learned better than the 20 people who reread the textbook
  - Not so impressive if we learn those 20 people compared notes outside of the experiment
- They will all do well or do poorly
Problem 1: Multiple Random Effects

• Important!
  • Also not so impressive if the 20 Practice Test subjects were all in the same biology section and the 20 Restudy subjects were in a different section
  • Need to account for differences in instructor, time of day
Independence assumption is fair if we randomly sample 1 person at a time.

- e.g., you recruit 40 undergrads from the Psychology Subject Pool.
- But maybe this isn’t all we should be doing… (Henrich et al., 2010, Nature)
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But many sensible, informative research designs involve more complex sampling procedures.

Example: Sampling multiple children from the same family
- Kids from the same family will be more similar
- a/k/a clustering
Problem 1A: Nested Random Effects

- But many sensible, informative research designs involve more complex sampling procedures
- Or: Kids in classrooms in schools
  - Kids from the same school will be more similar
  - Kids in same classroom will be even more similar!
One way to describe what’s going on here is that there are several levels of sampling, each **nested** inside each other.

- **SAMPLED SCHOOLS**
- **SAMPLED CLASSROOMS** in those schools
- **SAMPLED STUDENTS** in those classrooms

Each level is what we’ll call a **random effect** (a thing we sampled).
Two challenges:
- Statistically, we need to take account for this non-independence (similarity)
- Even a small amount of non-independence can lead to spurious findings (Quené & van den Bergh, 2008)
- We might want to characterize differences at each level!
- Are classroom differences or school differences bigger?
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Problem 1B: Crossed Random

• A closely related problem shows up in many experimental studies
• Experimental / research materials are often sampled out of population of possible items
  • Words or sentences
  • Educational materials
  • Hypothetical scenarios
  • Survey items
  • Faces
Problem 1B: Crossed Random Effects

We might ask:
- Do differences in stimuli used account for group / condition differences?
  - e.g., Maybe easier vocab words used in one condition

Maintenance rehearsal

Elaborative rehearsal

2. Wikipedia is usually my first resource for research.
**Problem 1B: Crossed Random Effects**

- We might ask:
  - Do differences in stimuli used account for group / condition differences?
  - Do our results *generalize* to the population of all relevant items?
  - All Spanish vocab words
  - All fictional resumes
  - All questionnaire items that measure extraversion
  - All faces
Problem 1B: Crossed Random Effects

- Again, we are sampling two things—subjects and items
- Arrangement is slightly different because each subject gets each item
  - Crossed random effects
- Still, problem is that we have multiple random effects (things being sampled)
Robustness across stimuli has been a major concern in psycholinguistics for a long time.
• Robustness across stimuli has been a major concern in psycholinguistics for a long time
  • If you are doing research related to language processing, you’ll be expected to address this
    • (But stats classes don’t always teach you how)

• Now growing interest in other fields, too

Treating Stimuli as a Random Factor in Social Psychology: A New and Comprehensive Solution to a Pervasive but Largely Ignored Problem

Charles M. Judd and Jacob Westfall
University of Colorado Boulder

David A. Kenny
University of Connecticut
Robustness across stimuli has been a major concern in psycholinguistics for a long time.

- If you are doing research related to language processing, you’ll be *expected* to address this.
  - (But stats classes don’t always teach you how)

Now growing interest in other fields, too.

- Be a statistical pioneer!
  - *Test* generalization across items
  - *Characterize* variability
  - *Increase power* (more likely to detect a significant effect)
Problem 1B: Crossed Random Effects

- OLD ANOVA solution: Do 2 analyses
  - Subjects analysis: Compare each subject (averaging over all of the items)
    - Does the effect generalize across subjects?
  - Items analysis: Compare each item (averaging over all of the subjects)
    - Does the effect generalize across items?

Note: not real data

SUBJECT ANALYSIS

\[ F_1(1,3) = 18.31, \ p < .05 \]

<table>
<thead>
<tr>
<th>Subject</th>
<th>Control</th>
<th>Primed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scott</td>
<td>1305</td>
<td>1122</td>
</tr>
<tr>
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<td>905</td>
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<tr>
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<td>900</td>
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<td>988</td>
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ITEM ANALYSIS

\[ F_2(1,4) = 22.45, \ p < .01 \]

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<th>Item</th>
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<tr>
<td>Knight</td>
<td>1100</td>
<td>883</td>
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<tr>
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<td>930</td>
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<tr>
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<td>912</td>
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<tr>
<td>Vacation</td>
<td>1094</td>
<td>830</td>
</tr>
<tr>
<td>Pirate</td>
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Problem 1B: Crossed Random Effects

- **OLD ANOVA solution:**
  - Do 2 analyses
    - **Subjects analysis**
    - **Items analysis**

- **Problem:** We now have 2 different sets of results. Might conflict!
  - Possible to combine them with $\text{min } F'$, but not widely used

### Subject Analysis

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$F_1(1,3) = 18.31, p < .05$

### Item Analysis

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Problem 2: Categorical Data

- ANOVA assumes our response is continuous
- But, we often want to look at categorical data

- Does student graduate high school or not?
- Item recalled or not?
- What predicts diagnosis of ASD?
Problem 1: Categorical Data

- Traditional solution: Analyze proportions
  - Maybe with some transformation (e.g., arcsine, logit)
- Violates assumptions of ANOVA
  - Among other issues: ANOVA assumes normal distribution, which has infinite tails
  - But proportions are clearly bounded
  - Model could predict impossible values like 110%

Problem 2: Categorical Data

- High School Graduation Rates, 2008-2009
  - Rural: 80%
  - Suburban: 81%
  - Urban: 67%

0 ≤ proportions ≤ 1
Problem 2: Categorical Data

- Traditional solution: Analyze proportions

  i need you to start giving me 110%
  no problem... i've worked 44-hour weeks before

- Model could predict impossible values like 110%

<table>
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But $0 \leq \text{proportions} \leq 1$
Problem 2: Categorical Data

- Traditional solution: Analyze proportions
  - Maybe with some transformation (e.g., arcsine, logit)
- Violates assumptions of ANOVA
- Can lead to:
  - Spurious effects (false positives / Type I error)
  - Missing real effects (false negatives / Type II error)

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Problem 3: Continuous Predictors

- Many interesting independent variables vary continuously
  - e.g., Word frequency

pitohui  penguin  eagle
Problem 3: Continuous Predictors

- Many interesting independent variables vary continuously
  - Or: Second language proficiency or reading skill

- ANOVAs require division into categories
  - e.g., median split
Many interesting independent variables vary continuously
  - Or: Second language proficiency or reading skill

ANOVAAs require division into categories
  - e.g., median split
  - Or: extreme groups design
Many interesting independent variables vary continuously
   - Or: Second language proficiency or reading skill

ANOVAAs require division into categories
   - Problem: Can only ask “is there a difference?”, not form of relationship
   - Loss of statistical power (Cohen, 1983)
Course goals & requirements

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Big picture view of mixed effects models

Terminology

R

Power of subjects analysis!

Power of items analysis!

Captain Mixed Effects to the rescue!
Mixed Effects Models to the Rescue!

- Biggest contribution of mixed-effects models is to incorporate *multiple random effects* into the *same analysis*

How does the effect of parental stress on screen time generalize across *children* and *families*?

How does the effect of aphasia on sentence processing generalize across *subjects* and *sentences*?
Mixed Models to the Rescue!

- Model outcome using regression-like approach

\[ \text{GPA} = \text{Motivational intervention} + \text{Subject} + \text{School} \]

Problem 1A solved!
Mixed Effects Models to the Rescue!

- For many experimental designs, this means a change in what gets analyzed
- **ANOVA**: Unit of analysis is *cell mean*

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<td>960</td>
<td>1214</td>
</tr>
<tr>
<td>Ben</td>
<td>913</td>
<td>1003</td>
</tr>
</tbody>
</table>

- **Mixed effects models**: Unit of analysis is *individual trial!*

<table>
<thead>
<tr>
<th>RT</th>
<th>PRIME?</th>
<th>SUBJECT</th>
<th>ITEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1224</td>
<td>Yes</td>
<td>Scott</td>
<td>eagle</td>
</tr>
<tr>
<td>683</td>
<td>No</td>
<td>Scott</td>
<td>penguin</td>
</tr>
<tr>
<td>432</td>
<td>Yes</td>
<td>Scott</td>
<td>pitohui</td>
</tr>
<tr>
<td>892</td>
<td>No</td>
<td>Scott</td>
<td>robin</td>
</tr>
<tr>
<td>1028</td>
<td>Yes</td>
<td>Ben</td>
<td>penguin</td>
</tr>
</tbody>
</table>
Mixed Models to the Rescue!

- Model outcome using regression-like approach
- Look at *individual trials/observations* (not means)

Semantic categorization: Is it a bird?

Problem 1B solved!
Mixed Models to the Rescue!

- In a regression, easy to include independent variables that are continuous

\[ \text{RT} = \text{Lexical Freq.} + \text{Subject} + \text{Item} \]

Problem 3 solved!
Mixed Models to the Rescue!

- Link functions allow us to relate model to DV that isn’t normally distributed

Accuracy

Odds of correct response on this trial (yes or no?)

= Lexical Freq. + Subject + Item

Problem 2 solved!
Mixed Models to the Rescue!

- Link functions allow us to relate model to DV that isn’t normally distributed

Odds of graduating college = Motivational intervention + Subject + School

Problem 2 solved!
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A Terminological Note…

- “Mixed effects models” is not the most precise term
  - Technically, any model that includes subjects, classrooms, or items (a “random effect”) plus experimental variables (“fixed effects”)

- Models we’ll be talking about are hierarchical linear models

- But “mixed effects models” has caught on in cognitive psychology
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How do we run mixed effects models?
- Multiple software packages could be used to fit the same conceptual model
- Most popular solution: R with lme4
<table>
<thead>
<tr>
<th><strong>R Pros</strong></th>
<th><strong>R Cons</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Free!</td>
<td>Documentation / help files not the best</td>
</tr>
<tr>
<td>Runs on any computer</td>
<td>Other online resources</td>
</tr>
<tr>
<td>Lots of add-ons—can do just about any type of model</td>
<td>Requires some programming, not just menus</td>
</tr>
<tr>
<td>Gaining popularity</td>
<td></td>
</tr>
<tr>
<td>Makes analyses clear &amp; more reproducible</td>
<td></td>
</tr>
</tbody>
</table>
Two Ways to Use R

- Regular R <www.r-project.org>
- RStudio <www.r-studio.com>
  - Different interface
    - Some additional windows/tools to help you keep track of what you’re doing
  - Same commands, same results
  - Also available for just about any platform
  - Recommended (but not required)
  - Requires you to download regular R first
- Also R Commander with buttons, menus
  - No experience with this, not sure if it works with lme4
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Wrap-Up

- Mixed effects models solve three common problems with ANOVAs
  - Multiple random effects (subjects, items, classrooms, schools)
  - Categorical outcomes
  - Continuous predictors
- For next week: Download R!
  - Next class, we’ll get started using R
- If sitting in, e-mail me (sfraundo@pitt.edu) for CourseWeb access
- CourseWeb survey about your research & statistical background