Course Business

• New R package to install: misty

• On Canvas for this week:
  • New dataset we’ll cover in lecture: numerosity
  • Lab materials

• Follow-up on last Wednesday
  • Thanks for working with change in schedule
  • Apologies for not having a larger lab available
Week 7.1: Centering & Transformations

- Centering
  - Today’s Dataset
    - Mean Centering
    - Centering Around Other Values
    - Logarithmic Transformation
    - Grand-Mean vs. Cluster-Mean Centering

- Lab
Today’s Dataset

- Brain warmup – count the number of dots
Today’s Dataset

- **numerosity.csv**: Looking at relation of **math anxiety** to basic number skills
  - Participants log in once a day on their smartphone to complete a dot-counting task
  - And, rate their math anxiety each time
  - 30 trials (one per day for a month)

- **Measures:**
  - RT for each trial
  - NumDots in each display
  - Math Anxiety on a scale of 1 to 7

<table>
<thead>
<tr>
<th>Subject</th>
<th>NumDots</th>
<th>Anxiety</th>
<th>RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>30</td>
<td>Min. 16.00</td>
<td>Min. 1.000</td>
</tr>
<tr>
<td>S10</td>
<td>30</td>
<td>1st Qu. 58.00</td>
<td>1st Qu. 1.000</td>
</tr>
<tr>
<td>S11</td>
<td>30</td>
<td>Median 66.50</td>
<td>Median 2.000</td>
</tr>
<tr>
<td>S12</td>
<td>30</td>
<td>Mean 67.33</td>
<td>Mean 2.236</td>
</tr>
<tr>
<td>S13</td>
<td>30</td>
<td>3rd Qu. 85.00</td>
<td>3rd Qu. 3.000</td>
</tr>
<tr>
<td>S14</td>
<td>30</td>
<td>Max. 102.00</td>
<td>Max. 6.000</td>
</tr>
<tr>
<td>(Other)</td>
<td>720</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Week 7.1: Centering & Transformations

- Centering
  - Today’s Dataset
  - Mean Centering
    - Centering Around Other Values
    - Logarithmic Transformation
    - Grand-Mean vs. Cluster-Mean Centering

- Lab
Interpreting Intercepts

• Let’s start with examining the effect of numerosity (NumDots) on RT in the dot-counting task
  • At this stage, we don’t care about Anxiety
    • Common that spreadsheet contains extra, irrelevant columns

• What would the maximal model for this be?
  • dotModel.Maximal <- lmer(RT ~ 1 + NumDots +
    data = numerosity)

• Hint #1: NumDots is a within-subjects variable

• Hint #2: Could there be a different effect of NumDots for each subject?
Interpreting Intercepts

• Let’s start with examining the effect of numerosity (NumDots) on RT in the dot-counting task
  • At this stage, we don’t care about Anxiety
    • Common that spreadsheet contains extra, irrelevant columns

• What would the maximal model for this be?
  • dotModel.Maximal <- lmer(RT ~ 1 + NumDots + (1 + NumDots | Subject) +
    data = numerosity)
  • Numerosity is manipulated within subjects (each subject sees several different display sizes)
    • Possible to calculate each subject’s personal NumDots effect (slope)—some subjects could count faster than others
  • Include random slope
Interpreting Intercepts

Let’s start with examining the effect of numerosity \((\text{NumDots})\) on RT in the dot-counting task

- At this stage, we don’t care about Anxiety
  - Common that spreadsheet contains extra, irrelevant columns

Results:

\[
y = 1690 + 9.4 \times \text{NumDots}
\]

- Intercept: RT is \textbf{1690} ms when number of dots is 0
- NumDots effect: +\textbf{9.4} ms for each dot
  - But, display size of 0 is \textit{impossible}. Odd to talk about.
Interpreting Intercepts

- Let’s change the model so that 0 means something

RT = 1690 ms + 8 * NumDots

Intercept: 1690 ms
Mean Centering

- Mean number of dots is ~67

- Imagine we subtracted this mean size from each display size

<table>
<thead>
<tr>
<th>Original</th>
<th>Mean Subtracted</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>35</td>
</tr>
<tr>
<td>67</td>
<td>0</td>
</tr>
<tr>
<td>58</td>
<td>-9</td>
</tr>
</tbody>
</table>

- New zero represents mean display size
  - “Mean centering”
Mean Centering

- **New zero** represents **mean** display size
- “**Mean centering**”
Centering—How to Do It

First, create a new variable:
- `library(misty)`
- `numerosity %>% mutate(NumDots.cen = center(NumDots))` -> `numerosity`

Then, use the new variable in your model
- `dotModel.cen.Maximal <- lmer(RT ~ 1 + NumDots.cen + (1 + NumDots.cen | Subject) + data = numerosity)`
Centering—Results

• Old:

| Fixed effects: | Estimate | Std. Error | df | t value | Pr(>|t|) |
|----------------|----------|------------|----|---------|-----------|
| (Intercept)    | 1000.421 | 152.547    | 24.800 | 11.081 | 4.28e-11 *** |
| NumDots        | 9.401    | 1.178      | 27.562 | 7.984  | 1.21e-08 *** |

---

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:

(Intr)

NumDots -0.499

• What hasn’t changed?
  • NumDots effect is still ~9.4 per dot

• What has changed?
  • Intercept is 2323 ms at mean numerosity
  • Correlation of NumDots effect with intercept is now almost 0. Indicates that we centered correctly.
  • New model: \( y = 2323 + 9.4 \times \text{NumDots} \)
Centering—Results

- Intercept: RT is 2094 ms at mean display size
- NumDots effect: +8 ms for each additional dot
Which Do You Like Better?

UNCENTERED
- Good if zero is *meaningful*
- Years of study abroad, number of previous trials, number of missed classes…

CENTERED
- Good if zero is *not meaningful* or *not observed*
- Reduced correlation w/ intercept also helps with convergence (esp. in binomial models)
Which Do You Like Better?

- Both regression equations apply only to plausible number of dots
  - With raw number of dots, can’t have a numerosity of 0 or less
  - With centered number of dots, can’t have a value of -67 or less (0 minus the mean of 67)
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    - Grand-Mean vs. Cluster-Mean Centering

- Lab
Centering Around Other Values

• We could also make 0 correspond to some other sensible/useful value
  • The smallest logically possible value
    • `numerosity` %%
      `mutate(NumDots2 = center(NumDots, value = 1))` -> `numerosity`

• The smallest *observed* value in our data
  • `numerosity` %>%
    `mutate(NumDots3 = center(NumDots, value = min(NumDots)))` -> `numerosity`
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Logarithmic Transformations

• Let’s look more closely at the relation between dots and RT…
  • Not totally linear…
Logarithmic Transformations

- Let’s look more closely at the relation between dots and RT...
  - Not totally linear...

Most points **above** the line at larger $x$ values
Most points **below** the line at small $x$ values
Logarithmic Transformations

• Let’s look more closely at the relation between dots and RT…
  • Not totally linear…

• Effect “levels off”
  • Effect of adding 1 more dot is smaller when there are already a lot of dots
  • “Diminishing returns”
Logarithmic Transformations

Fechner’s law
Logarithmic Transformations

• This type of non-linear relationship is well modeled by a logarithmic transformation

• `numerosity %>% mutate(NumDots.log = log(NumDots)) -> numerosity`
**Logarithmic Transformations**

- This looks more linear
- How can we compare the fit of these models?
  - Hint: We subtracted `NumDots` and added `NumDots.log`, so these are *not* nested models
Logarithmic Transformations

• This *looks* more linear
• How can we compare the fit of these models?
  • Hint: We subtracted *NumDots* and added *NumDots.log*, so these are *not* nested models

• `anova(dotModel.Maximal, dotModel.log.Maximal)`

<table>
<thead>
<tr>
<th>Model</th>
<th>npar</th>
<th>AIC</th>
<th>BIC</th>
<th>logLik deviance</th>
</tr>
</thead>
<tbody>
<tr>
<td>dotModel.Maximal</td>
<td>6</td>
<td>14066</td>
<td>14095</td>
<td>-7027.2</td>
</tr>
<tr>
<td>dotModel.log.Maximal</td>
<td>6</td>
<td>14053</td>
<td>14082</td>
<td>-7020.7</td>
</tr>
</tbody>
</table>

Model with log(*NumDots*) has lower (better) AIC and BIC
Logarithmic Transformations

- Logarithmic relationships pervasive
  - Many aspects of perception (Fechner and Weber laws)
  - Many aspects of experience
  - Dose-response
  - Income

<table>
<thead>
<tr>
<th>Source</th>
<th>Intensity</th>
<th>Intensity level</th>
<th>× TOH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold of hearing (TOH)</td>
<td>$10^{-12}$</td>
<td>0 dB</td>
<td>1</td>
</tr>
<tr>
<td>Whisper</td>
<td>$10^{-10}$</td>
<td>20 dB</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Pianissimo</td>
<td>$10^{-8}$</td>
<td>40 dB</td>
<td>$10^4$</td>
</tr>
<tr>
<td>Normal conversation</td>
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<tr>
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<td>100 dB</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>Threshold of pain</td>
<td>10</td>
<td>130 dB</td>
<td>$10^{13}$</td>
</tr>
<tr>
<td>Jet take-off</td>
<td>$10^2$</td>
<td>140 dB</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>Instant perforation of eardrum</td>
<td>$10^4$</td>
<td>160 dB</td>
<td>$10^{16}$</td>
</tr>
</tbody>
</table>

Table 1.1 from (Müller, FMP, Springer 2015)
Logarithmic Transformations

- Logarithmic relationships pervasive
  - Many aspects of perception (Fechner and Weber laws)
  - Many aspects of experience
  - Dose-response
  - Income

- If you want to center, apply the log transform first, then center
  - Otherwise, the final variable will not be centered
  - Idea is that the “true” variable has a log scale, so we want to center around that mean

<table>
<thead>
<tr>
<th>Source</th>
<th>Intensity</th>
<th>intensity level</th>
<th>$x$</th>
<th>TOH</th>
</tr>
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<tr>
<td>Threshold of hearing (TOH)</td>
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Table 1.1 from [Müller, FMP, Springer 2015]
Logarithmic Transformations

• When to apply a transformation?

• We have an *a priori* reason to expect a logarithmic (or other) relationship
  • e.g., some well-studied variable, like loudness or word frequency

• Empirically based on the observed relationship to the DV
  • But beware of overfitting!
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Let’s turn to the effects of math anxiety
Remember that people provided a different Anxiety rating on each of the 30 days (trials)
Is this a level-1 or level-2 variable?
Grand-Mean vs. Cluster-Mean Centering

- What mean to use to center a level-1 variable?
  - Grand mean?: 2.23
Grand-Mean vs. Cluster-Mean Centering

- What mean to use to center a level-1 variable?
  - Grand mean?: 2.23
  - But individual subject means differ a lot…
  - Subject 1: 1.97, Subject 11: 5.03, Subject 2: 4.03
Grand-Mean vs. Cluster-Mean Centering

• Both sources of variation may be relevant

  • Grand-mean centering: Are you more or less math-anxious compared to the average person (2.23)?
    • Trait anxiety

  • Cluster-mean centering: Are you more or less math-anxious on this day compared to your average (for subject 2: 4.03)?
    • State anxiety
Grand-Mean vs. Cluster-Mean Centering

• Centering a level-1 variable “smushes” together these two sources of variance (Hoffman, 2015, 2019)
  • How the cluster differs from the grand mean
  • How the observation differs from the cluster mean

• A better approach is to track these two influences separately
  • `numerosity %>% mutate(TraitAnxiety = center(Anxiety, type='CGM', group=Subject))` -> numerosity
  • Center around the grand mean (a level-2 variable)
  • `numerosity %>% mutate(StateAnxiety = center(Anxiety, type='CWC', group=Subject))` -> numerosity
  • Center around the cluster mean (a level-1 variable)
Then, include both of these in the model

```r
model.Anxiety <- lmer(RT ~ 1 + TraitAnxiety + StateAnxiety + (1|Subject), data=numerosity)
model.Anxiety %>% summary()
```

Can also include a random slope for the level-1 variable (StateAnxiety)
Grand-Mean vs. Cluster-Mean Centering

• Possible to have one effect, but not the other
  • Trait effect but not a state effect
    • Could reflect another individual difference (math skill?)

![Graph showing mean RT vs. anxiety for different subjects.](image)
Grand-Mean vs. Cluster-Mean Centering

- Possible to have one effect, but not the other
  - **Trait effect** but not a state effect
    - Could reflect another individual difference (math skill?)
  - **State effect** but not a trait effect
    - What matters is where you are relative to your norms

![Diagram showing correlation within a person and location of the person mean does not matter.](image)
• Possible to have one effect, but not the other
  • **Trait effect** but not a state effect
    • Could reflect another individual difference (math skill?)
  • **State effect** but not a trait effect
    • What matters is where you are relative to your norms
  • Or **both**!

*Grand-Mean vs. Cluster-Mean Centering*

People with more trait anxiety have longer RTs
Grand-Mean vs. Cluster-Mean Centering

• Conclusions:
  • For level-1 variables, want to consider both between- and within- cluster variation
  • This does not matter:
    • For level-2 variables (e.g., no within-person slope for IQ or math class GPA)
      • No within-cluster variance
    • Every cluster has the same mean (e.g., experimental manipulation like NumDots)
      • No between-cluster variance
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