Week 10.1: Empirical Logit

- Finish Generalized LMERs
  - Clarification on Logit Models
    - Poisson Regression
- Empirical Logit
  - Low & High Probabilities
  - Empirical Logit
  - Implementation
- Lab
Clarification on Logit Models

- Dependent/outcome variable would be a variable made up of 0s and 1s
  - We don’t need to apply any kind of transformation
  - `glmer(Recalled ~ 1 + StudyTime * Strategy + (1|Subject) + (1|WordPair), data=cuedrecall, family=binomial)`
  - `family=binomial` tells R to analyze this using the logit link function—everything handled automatically
Clarification on Logit Models

- When we get our results, they will be in terms of log odds/logits (because we used family=binomial)
- We may want to use \( \exp() \) to transform our estimates into effects on the odds to make them more interpretable
  - Elaborative rehearsal = +2.29 log odds of recall
  - \( \exp(2.29) = \) Odds of recall 9.87 times greater
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**Poisson Regression**

- `glmer()` supports other non-normal distributions, such as `family=poisson`.
- Good when the DV is a frequency count:
  - *Number of gestures made in communicative task*
  - *Number of traffic accidents*
  - *Number of ideas brainstormed*
  - *Number of doctor’s visits*

- Different from both normal & binomial distributions:
  - Lower-bound at 0
  - But, *no* upper bound
  - This is a Poisson distribution!
**Poisson Regression**

- **Link function** for a Poisson distribution is the $\log()$
- Thus, we would also want to use $\exp()$
  - Effect of ASD on number of disfluent pauses: $0.27$
  - $\exp(0.27) = 1.31$
  - ASD increases frequency of pauses by 1.31 times

\[
\log(y_i) = \gamma_0 + \gamma_1 X_{1i} + e_i
\]

- Can be any number
- Can be any number

- Intercept (Baseline)
- ASD
Poisson Models: Other Variants

- **Offset term**: Controls for differences in the opportunities to observe the DV (e.g., *time*)
  - e.g., one child observed for 15 minutes, another for 14 minutes
  - `glmer(Disfluencies ~ 1 + ASD + (1|Family), offset=Time, family=poisson)`
Poisson Models: Other Variants

- **Zero-inflated Poisson**: Sometimes, more 0s than expected under a true Poisson distribution.
  - Often, when a separate process creates 0s.
    - # of traffic violations → 0 if you don’t have a car.
    - # of alcoholic drinks per week → 0 if you don’t drink.

- Zero-inflated model: Simultaneously models the 0-creating process as a logit and the rest as a Poisson.

- Use package `zeroinfl`.
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Source Confusion

- Remember our cued recall data from last week?
- A second categorical DV here is **source confusions**:
  - Lots of theoretical interest in **source memory**—memory for the context or source where you learned something

  **Study:**
  - VIKING—COLLEGE
  - SCOTCH—VODKA

  **Test:**
  - VIKING—**vodka**

  *Odds are not the same thing as probabilities!*
Source Confusion Model

• Let’s model what causes people to make source confusions:
  • `model.Source <- lmer(SourceConfusion ~ Strategy*StudyTime.cen + (1|Subject) + (1|WordPair), data=sourceconfusion)`
  • This code contains two mistakes—can we fix them?
Let's model what causes people to make source confusions:

\[
\text{model.Source} \leftarrow \text{glmer}(\text{SourceConfusion} \sim \text{Strategy} \ast \text{StudyTime.cen} + (1|\text{Subject}) + (1|\text{WordPair}), \text{data}=\text{sourceconfusion}, \text{family}=\text{binomial})
\]

Failed to converge, with very questionable results

\[
\exp(20.08) = 525,941,212 \text{ times more likely in Maintenance condition}
\]

... but not significant
Low & High Probabilities

• Problem: These are low frequency events

• In fact, lots of theoretically interesting things have low frequency
  • Clinical diagnoses that are not common
  • Various kinds of cognitive errors
    • Language production, memory, language comprehension…
  • Learners’ errors in math or other educational domains
Low & High Probabilities

• A problem for our model:
  • Model was trying to find the odds of making a source confusion within each study condition
  • But: Source confusions were never observed with elaborative rehearsal, ever!
  • How small are the odds? They are infinitely small in this dataset!

• Note that not all failures to converge reflect low frequency events. But when very low frequencies exist, they are likely to cause convergence problems.
Low & High Probabilities

• Logit is **undefined** if probability = 1

\[
\text{logit} = \log \left[ \frac{p(\text{confusion})}{1-p(\text{confusion})} \right] = \log \left[ \frac{1}{0} \right]
\]

• Logit is also **undefined** if probability = 0

\[
\text{logit} = \log \left[ \frac{p(\text{confusion})}{1-p(\text{confusion})} \right] = \log \left[ \frac{0}{1} \right] = \log (0)
\]

• Log 0 is undefined
• \(e^{??} = 0\)
• But there is nothing to which you can raise \(e\) to get 0
Low & High Probabilities

- When close to 0 or 1, logit is defined but unstable

![Graph showing the relationship between probability of recall and log odds of recall.](image)

- Relatively gradual change at moderate probabilities
- Fast change at extreme probabilities

- $p(0.6) \rightarrow 0.41$
- $p(0.8) \rightarrow 1.39$
- $p(0.95) \rightarrow 2.94$
- $p(0.98) \rightarrow 3.89$
Low & High Probabilities

• A problem for our model:
  • Question was how much less common source confusions become with elaborative rehearsal
  • But: Source confusions were never observed with elaborative rehearsal

• Why we think this happened:
  • In theory, elaborative subjects would probably make at least one of these errors eventually (given infinite trials)
    • Not impossible
  • But, empirically, probability was low enough that we didn’t see the error in our sample (limited sample size)
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Empirical Logit

• **Empirical logit**: An adjustment to the regular logit to deal with probabilities near (or at) 0 or 1

\[
\text{logit} = \log \left[ \frac{p(\text{confusion})}{1-p(\text{confusion})} \right]
\]
Empirical Logit

- **Empirical logit**: An adjustment to the regular logit to deal with probabilities near (or at) 0 or 1

$\text{logit} = \log \left( \frac{\text{Num of "A"s}}{\text{Num of "B"s}} \right)$

$\text{emp. logit} = \log \left( \frac{\text{Num of "A"s} + 0.5}{\text{Num of "B"s} + 0.5} \right)$

- Makes extreme values (close to 0 or 1) less extreme

$A = \text{Source confusion occurred}$

$B = \text{Source confusion did not occur}$
Empirical Logit

- **Empirical logit**: An adjustment to the regular logit to deal with probabilities near (or at) 0 or 1

\[
\text{logit} = \log \left( \frac{\text{Num of “A”s}}{\text{Num of “B”s}} \right)
\]

\[
\text{emp. logit} = \log \left( \frac{\text{Num of “A”s + 0.5}}{\text{Num of “B”s + 0.5}} \right)
\]

- Makes extreme values (close to 0 or 1) less extreme
Empirical logit doesn’t go as high or as low as the “true” logit.

At moderate values, they’re essentially the same.
With larger samples, difference gets much smaller (as long as probability isn’t 0 or 1)
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Empirical Logit: Implementation

- Empirical logit requires summing up events and then adding 0.5 to numerator & denominator:

\[
\text{empirical logit} = \log \left[ \frac{\text{Num } A + 0.5}{\text{Num } B + 0.5} \right]
\]

- Thus, we have to (1) sum across individual trials, and then (2) calculate empirical logit.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Strategy</th>
<th>StudyTime</th>
<th>NumSourceConfusions</th>
<th>TotalTrials</th>
<th>EmpLogit</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Elaborative</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>-1.95</td>
</tr>
<tr>
<td>S1</td>
<td>Elaborative</td>
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<td>0</td>
<td>2</td>
<td>-1.61</td>
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<td>0</td>
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<td>-3.30</td>
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<td>0</td>
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<td>0</td>
<td>5</td>
<td>-2.40</td>
</tr>
<tr>
<td>S1</td>
<td>Elaborative</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>-1.61</td>
</tr>
</tbody>
</table>

Single empirical logit value for each subject in each condition
Not a sequence of one YES or NO for every item

Num of As for subject S10 in Low associative strength, Maintenance rehearsal condition
Empirical Logit: Implementation

- Empirical logit requires summing up events and then adding 0.5 to numerator & denominator:

\[
\text{empirical logit} = \log \left[ \frac{\text{Num } A + 0.5}{\text{Num } B + 0.5} \right]
\]

- Thus, we have to (1) sum across individual trials, and then (2) calculate empirical logit.
- **Can’t** have multiple random effects with empirical logit.
  - Would have to do separate by-subjects and by-items analyses.
- Collecting more data can be another solution.

Num of As for subject S10 in Maintenance rehearsal condition.
Empirical Logit: Implementation

- Scott’s `psycholing` package can help calculate the empirical logit & run the model
- Example script will be posted on Canvas

- Two notes:
  - No longer using `glmer()` with `family=binomial`. We’re now running the model on the empirical logit value, which isn’t just a 0 or 1.

Here, the value of the DV is -1.61
Empirical Logit: Implementation

• Scott’s psycholing package can help calculate the empirical logit & run the model
  • Example script will be posted on Canvas

• Two notes:
  • No longer using glmer() with family=binomial. We’re now running the model on the empirical logit value, which isn’t just a 0 or 1.
  • Because we calculate the empirical logit beforehand, model doesn’t know how many observations went into that value
    • Want to appropriately weight the model

-2.46 could be the average across 10 trials or across 100 trials
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