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Coordinating principles and examples through analogy and self-explanation

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Abstract Research on expertise suggests that a critical aspect of expert understanding is knowledge of the relations between domain principles and problem features. We investigated two instructional pathways hypothesized to facilitate students' learning of these relations when studying worked examples. The first path is through self-explaining how worked examples instantiate domain principles and the second is through analogical comparison of worked examples. We compared both of these pathways to a third instructional path where students read worked examples and solved practice problems. Students in an introductory physics class were randomly assigned to one of three worked example conditions (reading, self-explanation, or analogy) when learning about rotational kinematics and then completed a set of problem solving and conceptual tests that measured near, intermediate, and far transfer. Students in the reading and self-explanation groups performed better than the analogy group on near transfer problems solved during the learning activities. However, this problem solving advantage was short lived as all three groups performed similarly on two intermediate transfer problems given at test. On the far transfer test, the self-explanation and analogy groups performed better than the reading group. These results are consistent with the idea that self-explanation and analogical comparison can facilitate conceptual learning without decrements to problem solving skills relative to a more traditional type of instruction in a classroom setting.

Keywords Analogy · Explanation · Generalization · Instruction · Learning · Principles · Problem solving · Knowledge transfer · Worked examples

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The central problem addressed in this work is how to facilitate students' conceptual understanding and problem solving skills in physics. One way to address this problem is to examine which knowledge components comprise "expert understanding" and then design learning environments to help novices construct that knowledge (see Dufresne et al. 1992, for a similar approach). Previous research on expertise has shown that when experts in domains such as chess and physics solve novel problems within that domain, they can identify the deep structure or principles of the problem and then use that knowledge to identify relevant problem features and execute a set of procedures appropriate for the task (Chase and Simon 1973; Chi et al. 1981; Larkin et al. 1980; see Nokes et al. 2010 for an overview). This research suggests that a key component of expert understanding is knowledge of the relations between domain principles and the problem features. If we can design learning activities to help students acquire these relations, this should improve their conceptual understanding and future problem solving, essentially moving students' knowledge state closer to that of an expert. Furthermore, knowledge of these relations should support students in transferring their conceptual knowledge to new problems with different contents than those encountered during learning (Chi and VanLehn 2012).

We investigated two activities hypothesized to facilitate the acquisition of the relations between principles and examples. The first is self-explaining how worked examples relate to the domain principles and the second is making analogical comparisons between worked examples. We compared these two activities to a third type in which students read worked examples and solved practice problems. Much prior work has shown that the study of worked examples is an effective learning practice compared to unaided problem solving (e.g., Paas and Van Merrienboer 1994; Cooper and Sweller 1987). However, research has also shown that it often leads to knowledge closely tied to the contents and contexts of learning (i.e., near transfer) unless students engage in constructive processing during study, such as self-explanation, to promote conceptual learning and far transfer (Chi et al. 1989; Renkl 1997). Therefore, we hypothesized that students' who self-explained or compared worked examples would learn more of the conceptual relations necessary to support intermediate and far transfer than students who read worked examples.

We tested these hypotheses in an "in vivo" experiment conducted in physics classes at the United States Naval Academy (USNA). To determine what was learned we created assessments based on the transfer taxonomy proposed by Barnett and Ceci (2002). In their taxonomy of far transfer tasks, Barnett and Ceci identified the transfer content (i.e., what is transferred) and context (i.e., when and where it is transferred) as the two central factors for determining the transfer distance between learning and test. Based on their analysis of the contents of transfer (i.e., nature of the learned skills/concepts and the memory demands of the task), we designed three types of physics assessments to measure near, intermediate, and far transfer from our learning materials. We defined near transfer as problems that required the execution of prior problem solving procedures, intermediate transfer as problems that required the use of prior concepts in order to correctly recognize, execute, and adapt prior problem solving procedures, and far transfer as problems that required the recognition and use of prior concepts and principles to reason about new problem features and relations. Before describing the experiment in detail, we first review the prior work relevant to the investigation of these hypotheses.

Prior work

Learning from principles and examples

Unlike experts, novices do not typically work forward from the domain principles to the solution, but instead, use backwards working strategies (e.g., means–ends analysis) and prior examples to solve novel problems (Anderson et al. 1981; Simon and Simon 1978; VanLehn 1998). Furthermore, laboratory studies indicate that students prefer to use examples even when they have access to written instructions or principles (LeFerve and Dixon 1986; Ross 1987). For example, LeFerve and Dixon (1986) showed that when learning to solve induction problems, students preferred to use the solution procedure illustrated in the example to the one described in the written instructions.

Although using examples enables novices to make progress when solving new problems, they are often only able to apply such knowledge to near transfer problems with similar surface features (see Reeves and Weissberg 1994, for a review). One reason that students may rely so heavily on prior examples to solve new problems is that they lack a deep understanding for how the principles of the domain are instantiated in the examples. That is, they may lack the knowledge and skills required to relate the relevant conceptual knowledge components to the problem features.

Research by Nisbett and colleagues (Fong et al. 1986; Fong and Nisbett 1991) has shown that when students are given brief training on an abstract rule (e.g., the statistical principle for the Law of Large Numbers) with illustrating examples, they perform better than students trained on the rule or examples alone. This result was shown in a domain where the students were hypothesized to have an intuitive understanding of the principle prior to training. One plausible interpretation of this result is that the students used their intuitive understanding of the principle to generate inferences relating the abstract rule to the illustrating examples. This possibility is intriguing and suggests that a training procedure designed to facilitate inferences connecting principles to examples may result in robust learning of the underlying concepts.

A second strand of research that demonstrates the potential power of linking examples to principles is the work on learning from worked examples. Worked examples are problems that include a step-by-step, "worked-out", solution. Each step of the solution is given and sometimes include instructional explanations or justifications for that step (we will describe this feature in detail in the "Instructional explanations" section). As mentioned above, worked examples have been shown to support learning, problem solving, and near transfer. For example, in a seminal study that compared students who studied worked examples to those who solved problems, the students who studied the examples performed better on subsequent problem solving tasks (Cooper and Sweller 1987). This early work inspired much research aimed at understanding the worked example effect on learning and transfer (for reviews see Atkinson et al. 2000; Renkl 2005, 2011; Sweller et al. 1998).

Worked examples are hypothesized to be effective for several reasons. First, they provide constraints on the solution space. That is, they highlight the correct solution path so students do not have to waste time on incorrect or unfruitful searches. Second, they are hypothesized to reduce irrelevant cognitive load by highlighting the important elements of the problem and solution for the learner to focus on, encode, and reason about (Paas and Van Merrienboer 1994; Ward and Sweller 1990). Third, they are hypothesized to encourage constructive cognitive processes such as self-explanation in which the learner can explain to herself the underlying conceptual logic and justification behind each step (Catrambone 1998; Renkl 1997). It is for this last reason that we think worked examples may be particularly important for helping students link the features of an example to the abstract features of the underlying principle or concepts.

We used worked examples in all three of our instructional conditions. The content of the examples was the same across the conditions and included a problem description, solution steps, and instructional explanations. The examples were also interspersed with problem solving tasks during the learning phase for all three conditions. Prior work has shown that worked examples are more effective when embedded in problem solving activities than when studied alone (Renkl et al. 2000; Ward and Sweller 1990; Wittwer and Renkl 2008). The primary differences between the conditions were the types of activities each condition received with the worked examples that either prompted self-explanation, analogical comparison, or reading with additional problem solving practice. Activities designed to focus students on coordinating the relations between examples and principles should improve their conceptual understanding and far transfer. We hypothesize that one such activity is learning from self-explanation.

Self-explanation

Research on self-explanation has shown it to be an effective and robust learning activity (see Chi 2000; Fonseca and Chi 2011; Roy and Chi 2005; Siegler 2002, for recent reviews). Self-explanation, or the process of explaining to oneself with the goal of making sense of new information, has been shown to be positively correlated with learning gains across a variety of domains including biology (Chi et al. 1994), computer programming (Pirolli and Recker 1994), mathematics (Siegler 2002; Rittle-Johnson 2006), and physics (Chi et al. 1989; Nokes et al. 2011).

Research on self-explanation has shown that students who self-explain are more likely to generate new information not explicitly communicated in the learning materials. For example, Chi et al. (1989) found that good learners studying worked examples in physics generated more inferences than poor learners. Not only did these good learners generate more inferences but they also generated explanations that refined or expanded the conditions for applying parts of the example solutions whereas the poor learners did not. In addition, the good learners were more likely than the poor learners to mention that they did not understand a particular part of the example. This result suggests that a key component of the self-explanation process is to identify something that one does not understand and then try to explain it by generating inferences and revising prior knowledge or misconceptions (Chi 2000). Self-explanation is not merely correlated with greater learning, but laboratory experiments have also documented a causal connection (Aleven and Koedinger 2002; Atkinson et al. 2003; Chi et al. 1994; Conati and VanLehn 2000; Schworm and Renkl 2007). For example, Chi et al. (1994) showed that students who were prompted to self-explain while reading a biology text about the functions and anatomy of the heart generated more inferences—and more *correct* inferences, e.g., regarding functions not explicitly mentioned in the text—than an unprompted group. The prompted group also acquired a deeper conceptual understanding of the learning material than the unprompted group as measured by improvement from pre to post-tests.

Of particular interest for the current experiment were some promising results from the Chi et al. (1989) study showing that good learners were more likely than poor learners to generate inferences relating the features of the worked examples to the relevant physics principles and concepts for those problems. Further support for this relationship was found in Renkl's (1997) study of individual differences in students' self-explanations when learning about probability from worked examples. He found that the number of principle-based self-explanations that students gave while studying worked examples was positively correlated with their later performance on a difficult posttest. A similar result was found by Neuman et al. (2000) that showed good problem solvers in mathematics were more likely to self-explain the justifications for procedural steps than poor problem solvers. Finally, Berthold et al. (2009) conducted an experiment in which students were given open prompts to self-explain, prompts with scaffolding to self-explain, or no prompts to self-explain while studying multirepresentational probability examples. They found that students who were prompted to self-explain generated more self-explanations and showed greater procedural and conceptual learning than those who were not prompted. Furthermore, self-explanations that connected the underlying principle rationale to the example features were associated with greater conceptual learning. Taken together, these four studies provide evidence that making connections between the features of the examples and the principles and concepts is related to high levels of learning and performance. In the current experiment, we build on these results and test whether prompting students to self-explain worked examples will promote learning these relations and far transfer. In the next section, we describe a second activity hypothesized to promote learning these relations, namely the acquisition of a schema through analogical comparison.

Schema learning through analogical comparison

A problem schema is a knowledge organization of the information associated with a particular problem category. Problem schemas typically include declarative knowledge of principles, concepts, and formulae, as well as the procedural knowledge for how to apply that declarative knowledge to solve a problem. Schemas have been hypothesized to be the representation underlying the organization of expert knowledge (Chase and Simon 1973; Chi et al. 1981; Larkin et al. 1980; Nokes et al. 2010). One way in which schemas can be acquired is through analogical comparison (Gick and Holyoak 1983). Analogical comparison consists of aligning and mapping two example problem representations to one another and then extracting their commonalities (Gentner 1983; Gick and Holyoak 1983; Hummel and Holyoak 2003). This process discards the elements that do not overlap between two examples but preserves the common elements. The resulting knowledge representation consists of fewer superficial similarities than the examples, but retains the deep causal structure of the problems.

Research on analogy and schema learning has shown that the acquisition of schematic knowledge promotes transfer to novel problems. Many researchers have found a positive relationship between the quality of the abstracted schema and transfer to a novel problem that is an instance of that schema (Catrambone and Holyoak 1989; Gick and Holyoak 1983; Novick and Holyoak 1991). For example, Gick and Holyoak (1983) found that transfer of a solution procedure was greater when participants' schemas contained more relevant structural features.

Analogical comparison has also been shown to improve learning even when both examples are not initially well understood (Kurtz et al. 2001; Gentner et al. 2003). By comparing the commonalities between two examples, students can focus on the causal structure and improve their learning about the concept. Specifically, by comparing the examples students are provided an opportunity to link those examples to the underlying principle. Kurtz et al. (2001) showed that students who were learning about the concept of heat transfer learned more when comparing examples than when studying each example separately.

There has been much recent work examining how students learn from comparing worked examples in mathematics and science learning (e.g., Seufert 2003; Richland and McDonough 2010; Rittle-Johnson and Star 2007; see Alfieri et al. 2012 and Guo et al. 2012, for overviews). In one recent review of this research, Guo et al. (2012) argued that the design of effective example comparisons required careful attention to both the differences and similarities across the examples in order to focus students on the "critical" features of the problems. Furthermore, they argued that the researchers should define the critical features in terms of what the students already know, because their prior knowledge will affect what they notice as similarities and differences across the examples. The authors then described a number of ways in which worked examples can be designed to scaffold such learning by using patterns of variance and invariance across the examples.

The current work addresses the question of how analogical comparison may help bridge students' learning of the relations between principles and examples. Specifically, by comparing worked examples students are provided an opportunity to focus on how the critical features of the problems relate to the domain principles. Of course, it is possible that students may not focus on the most important features when using analogy but instead may focus on the surface features or the irrelevant commonalities between the problems. Prior work has shown that poor learners often use analogies "blindly" by just copying over line by line equations from one problem to another (VanLehn and Jones 1993; VanLehn 1998). To avoid poor analogical reasoning strategies, we have designed the worked examples to have specific structures that the students can discover through comparison. Our design follows several of the principles highlighted by Guo et al. (2012) such as taking into account student knowledge when defining the critical features of the examples and varying surface features to focus students on the underlying common concepts. In addition, we provide students with scripted prompts that focus their comparisons on particular aspects of the problems (for a similar approach, see Gentner et al. 2003).

Instructional explanations

We compared prompting self-explanation and analogical comparison of worked examples to a third type of instruction in which students read worked examples with instructional explanations followed by problem solving practice. Instructional explanations provide the rationale and justification for the procedural steps within the worked example. They help to answer implicit questions of why a particular step was taken or why a particular concept or principle was applied (Leinhardt 2001). Our instructional explanations include definitions of concepts and symbols in the equations. After reading through two worked examples, students were given an opportunity to solve an isomorphic practice problem.

Prior research testing the efficacy of learning from instructional explanations in worked examples has shown mixed results, some positive (e.g., Atkinson 2002; Experiment 1) and some negative (e.g., Hilbert et al. 2004). A recent meta-analysis by Wittwer and Renkl (2010) revealed that across 21 studies instructional explanations in worked examples had only a minimal positive impact on learning (overall effect, $d=0.16$). Furthermore, they found that several study variables including the type of test, the domain, and the comparison condition had a moderating effect on the benefit of those explanations. Specifically, the analysis revealed that instructional explanations were beneficial on conceptual tests that measured students declarative knowledge of the concepts and principles but not on near and far transfer tests that measured whether students could solve problems that had either similar or different structures from the worked examples. Instructional explanations also had a stronger effect in the domain of math than science or instructional design. They were shown to be effective when compared to control conditions that did not include those explanations but not when compared to control conditions that encouraged self-explanation.

The authors argued that students given instructional explanations were more likely to acquire declarative knowledge of the key principles and operators from the worked examples than the students in the control conditions that did not encourage self-explanation and that this knowledge facilitated better performance on the conceptual assessments. The fact that the instructional explanations were not any more or less effective than conditions that encouraged self-explanation suggests that conditions that encouraged self-explanations also facilitated some conceptual learning. However, work by Hausmann and VanLehn (2007) has shown that self-explanation leads to more learning and far transfer than paraphrasing instructional explanations for students studying worked examples in physics. We hypothesize that declarative knowledge acquired from studying instructional explanations is not well connected to the problem features and therefore does not support transfer to solving new problems.

We chose reading instructional explanations as a comparison condition for two reasons. First, it provides a clear contrast in terms of the amount of observable activity students engage in while studying the worked examples. Giving self-explanations and making analogical comparisons requires actively generating responses to the prompts or questions, whereas reading instructional explanations can be performed more passively. This characterization is consistent with the active–constructive–interactive framework for classifying student behaviors and their related cognitive processes by Chi (2009). Students in the self-explanation and analogical comparison conditions are more likely to engage in constructive behaviors (e.g., explain or elaborate) whereas students in the reading condition are more likely to engage in active (e.g., paraphrasing) or passive behaviors. Although students in the reading condition were also given a practice problem to solve which required some knowledge generation, this activity is likely to focus them on the application and execution of the prior procedures from the worked example and less on the conceptual explanations and the relations between problem features and those explanations. Second, research on instructional explanations suggests that this path promotes more conceptual learning than worked examples without instructional explanations. Therefore, if we see performance improvements in comparison to this condition it will be relative to an already effective form of instruction suggesting large learning benefits compared to unaided problem solving.

Experiment

Science, technology, engineering, and mathematics instruction typically provides students with written descriptions of the abstract principles in the domain, worked examples, and practice problems. We examine whether self-explanation and analogical comparison of worked examples facilitates learning the relations between the abstract principles and example features compared to studying worked examples with instructional explanations and additional practice. To assess whether students learn these relations we have created assessments based on the taxonomy of far transfer by Barnett and Ceci (2002). In addition, we test whether the previous results found in laboratory scenarios generalize to an instructional intervention administered in a classroom setting. Further, very little work has directly contrasted the learning benefits from self-explanation and analogical comparison when students were learning from the same materials, in this case, worked examples (see Gadgil et al. 2012 for a similar contrast but in a different domain).

Although we hypothesize that both self-explanation and analogy activities will result in learning of the relations between the principles and examples, differences could emerge in the specifics of what is learned and how that knowledge is represented. For example, engaging in self-explanation is hypothesized to facilitate inference generation that supplements, corrects, or extends one’s prior knowledge about the target learning material (Chi 2000). The result is typically a deeper, more coherent, elaborated knowledge representation of the example or expository text that one is explaining (Chi et al. 1994).

In contrast, much of the work on analogical comparison has focused on the abstraction of relational information from multiple examples. This learning activity focuses on highlighting and abstracting the critical features across examples (e.g., Catrambone and Holyoak 1989). Although making analogical comparisons may involve producing explanations, such explanations would still be focused on constructing schemas and recognizing abstract relations between problems rather than within problems. Therefore, a hypothesis that emerges from comparing these two pathways is that analogical comparisons may lead to the construction of more abstract features than self-explanation whereas self-explanation may lead to a deeper, more coherent understanding of particular cases and their procedures.

To test these hypotheses, we conducted a classroom experiment in which students received one of three-instructional interventions when learning about rotational kinematics. The intervention took place during the laboratory recitation of their introductory physics course before they had received any classroom instruction on the topic. All students were given a principle booklet that defined, illustrated, and described seven target concepts in rotational kinematics. After reading through the booklet, participants were given the learning booklet for their instructional condition that consisted of six worked examples, arranged in three pairs.

As previously mentioned, the worked examples were the same across the three conditions. However, the activities each group engaged in with those examples differed. Students in the reading condition read the worked examples, which included instructional explanations and justifications for each solution step. After reading each pair of examples, they were given an isomorphic practice problem to solve. The students in the self-explanation group were given the same worked examples, but were instructed to generate their own explanations and justifications after which they read the same instructional explanations that the reading group had received. The students in the analogical comparison condition read through the worked examples and instructional explanations (like the reading group), but then completed a compare and contrast activity with each pair of examples. Students in all three conditions were given near transfer problems to solve after studying each pair of examples. Students were then given a test to measure intermediate and far transfer.

Predictions

We predicted that the reading and self-explanation groups would perform better than the analogy group on the near transfer problems during learning. The reading group should perform well on these tasks because they are very similar to the worked examples that they studied and the practice problems that they solved. We hypothesized that students in the reading group would acquire knowledge of what equations are associated with particular problem types, as well as procedural knowledge for how to implement those equations to find the quantitative solutions. This knowledge should transfer well to the near transfer problems that simply differ in the cover story. The students in the self-explanation group were also expected to perform well on the near transfer problems, because they practiced generating justifications for each step of the problem solution. We expected that this activity should lead to the construction of both conceptual knowledge for the justification of each step as well as procedural knowledge for the order and sequence of those steps. This procedural knowledge should also facilitate performance on near transfer problems that have the same structure and in which the same sequence of steps apply. Finally, we expected poorer performance on the near transfer for students in the analogy group because they were focused on comparing and contrasting features across the problems, and not on the sequence and interrelation of the procedural steps, nor on practicing the computations associated with those steps. Therefore, we expected this group to learn important conceptual features of the problems but to have more difficulty in applying that abstract knowledge to problem solving.

In contrast to the predictions on the near transfer problems, we expected both the self-explanation and analogy groups would perform better than the reading group on the intermediate and far transfer tasks. Knowledge of the conceptual relations between principles and examples should help these students recognize the critical problem features in the intermediate transfer problems in order to apply and adapt the prior procedures from the worked examples. We expected the analogy group to perform similarly to the self-explanation group for these problems because they rely on understanding the conceptual features in order to identify what prior procedures are required to solve the problem. On the far transfer test, we expected the self-explanation and analogy conditions to perform better than the reading condition because they could use their knowledge of the relations between the principles and examples to apply those concepts to new problems with different features.

Methods

Participants

Fifty-four students from the USNA participated as a part of their normal course work. The students comprised three sections of the introductory physics course taught by two faculty members at the USNA.[1] The number of students in each section ranged between 16 and 21 and the experiment took place during their normally scheduled lab time.

Design

A between-subjects design was used with students in each section randomly assigned to one of the three learning conditions: reading (n=18), self-explanation (n=18), and analogy (n=18). The lab session was 100 min in length and was divided into a learning and test phase. See Fig. 1 for an overview of the experiment design, materials, and procedure.

Materials

All materials were created by the research team and vetted by the physics learn lab course committee at the Pittsburgh Science of Learning Center (www.learnlab.org) which included several physics instructors at both the college and high school levels.

Learning materials All students received a principle booklet, a calculator, and a headset microphone with a digital audio recorder. Students were given audiorecorders so we could record their talk-aloud protocols as they went through the learning materials.[2] The principle booklet was seven pages long and gave a description of the principles and concepts of rotational motion including: angular displacement, angular velocity, angular acceleration, tangential velocity, tangential acceleration, and radial (centripetal) acceleration. The principles each had written descriptions, graphic illustrations, and formulae. See Fig. 2 for an example principle description of angular displacement. The descriptions were modeled after those typically presented in textbooks. In addition to the principle descriptions, the booklet consisted of a brief tutorial describing circular motion and the interrelationships between each of the concepts including equations and figures. Next, we describe the booklets that each condition received.

[1] A fourth section of 22 students was dropped due to an implementation error, they did not have enough time to finish the test phase of the experiment.

[2] After data collection, we randomly sampled a subset of students' verbal protocols from each condition and listened to whether they complied with the instructions. Those protocols showed that students engaged in the desired activities.

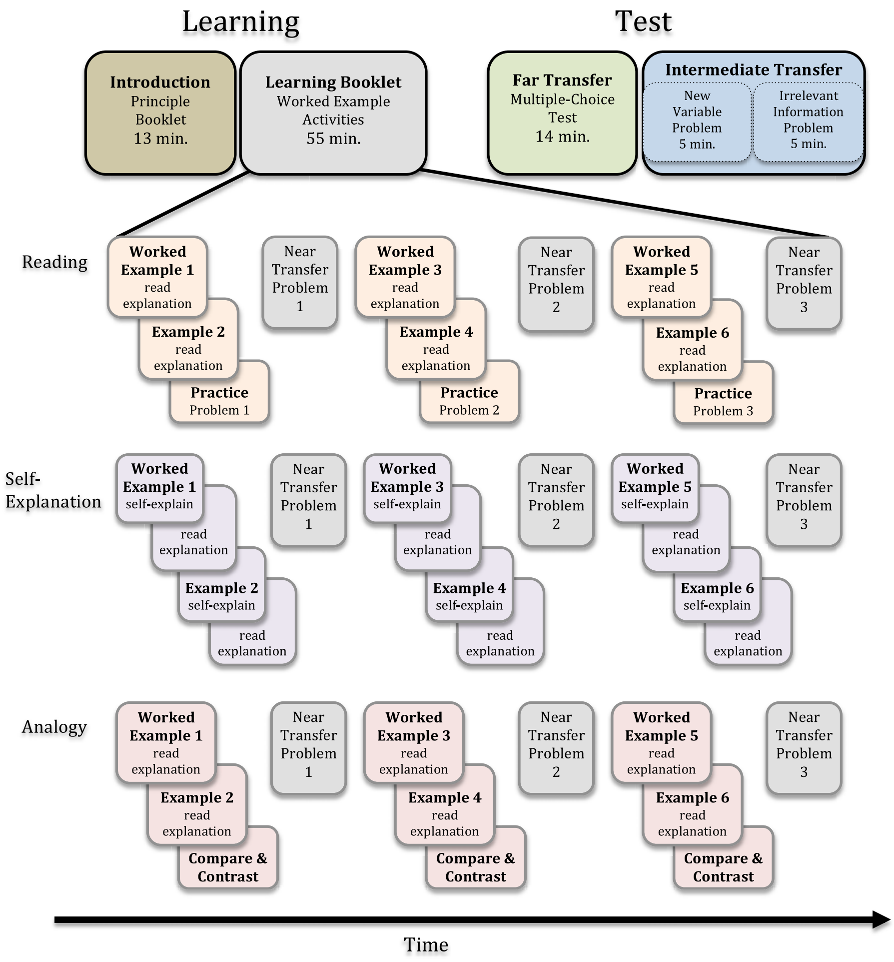

Fig. 1 Overview of the experiment design, materials, and procedure

Reading booklet The booklet for the reading condition consisted of a total of six worked examples (i.e., word problems with the step-by-step solutions) that included instructional explanations (see Fig. 3 for an illustration). Each example consisted of several parts. For instance, the example illustrated in Fig. 3 has three parts: (1) solving for angular velocity, (2) solving for radial acceleration, and (3) solving for tangential acceleration. The explanations (presented in the boxes) included principle definitions linking the symbols in the equations to the concepts and problem features in the worked example (e.g., angular velocity in regards to spaceships and orbits). The explanations also provided justifications for the steps in the problem. In addition, the explanations included descriptions of the relationships between concepts such as the relationship between angular velocity, angular acceleration, and tangential acceleration (see Fig. 3c).

For each example, students were told to read aloud for comprehension and understanding and to use the principle booklets to help them understand the materials. In addition, they were told that part of the solution for each worked example would be left blank and their task

Coordinating principles and examples

Principle Description

Angular Displacement: For a particle rotating in a circle, angular displacement is the angle in radians through which a point has been rotated about a specific axis (at the origin of a coordinate system). For perfectly circular motion, this angle is related to the arc length in inverse proportion to the radius of the circular path.

Symbolic Representation

Variable	**Meaning**	**Associated Equations**
θ	Angular Displacement	$\theta = \frac{s}{r}$ $\theta = \theta_0 + \omega t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ (constant acceleration)

Diagram for Pure Circular Motion

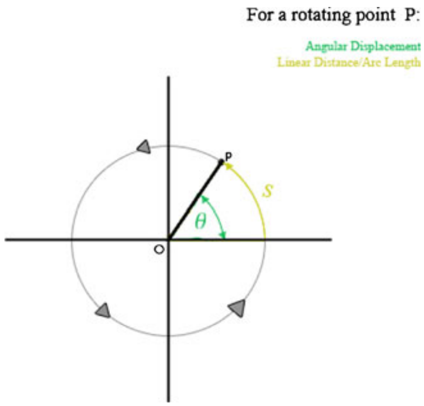

Fig. 2 Principle description of angular displacement

was to fill it in. To fill-in-the-blanks, students would have to find a quantity in the problem statement and then instantiate it in the equation. In the example above, it is to enter 18,000 mi/h for the velocity variable in the formula for part A. This was done to make sure the participants were attentively working through the problems.

After the presentation of the worked example, the next page of the booklet had the solutions to the fill in the blanks. This format was repeated for the next worked example in the learning booklet. After the second worked example and solution, the next page of the booklet had an isomorphic practice problem. This problem required the application of the same concepts and equations and could be solved via the same procedure as in the worked examples but had a different cover story. For example, instead of a spaceship negotiating a turn it was a toddlers' toy truck (see Fig. 4 for an example). We consider it a near transfer problem because it requires the same equations and procedures in the exact same order as the worked examples. Furthermore, students were told to use the prior example to help them solve it. After solving the practice problem, they could turn the page and see the solution. This problem was not given to the other conditions and served to help participants practice what they were learning as well as controlling for time on task (see Fig. 1). Reading examples and solving practice problems appears to be the modal learning activity in most physics courses (e.g., Serway and Jewett 2004).

1.1) A spaceship is negotiating a circular turn of radius 2000 mi at a constant speed of 18,000 mi/h. Assume the motion is counterclockwise, and that this is the positive direction.

(a) What is the angular velocity?

Givens: $r = 2000mi,\ v = 18000\,{}^{mi}\!/_{h}$

Sought: W_z

$$W_z = \frac{v}{r}$$

$$W_z = \frac{\quad\quad}{2000mi}$$

$$W_z = 9\,{}^{rad}\!/_{h}$$

The angular velocity (W_z) is the rate at which the angular position of the spaceship is changing as it moves in a circular orbit.

We choose the formula that relates angular velocity with translational speed (v), since we are given speed of the spaceship and radius of the orbit (r) in the statement of the problem.

(b) What is the radial acceleration?

Sought: a_R

$$a_R = W_z^2 r$$

$$a_R = (9\,{}^{rad}\!/_{h})^2(\underline{\quad\quad})$$

$$a_R = (.0025\,{}^{rad}\!/_{s})^2(\underline{\quad\quad})$$

$$a_R = 0.0125\,{}^{mi}\!/_{s^2}$$

The spaceship's radial acceleration (a_R) arises as a result of the constantly changing direction of its translational velocity.

Since we just derived angular velocity of the spaceship (W_z), we can use it, along with the radius of the orbit (r), to find the radial (centripetal) acceleration.

Before computing, we convert from hours to seconds in order to obtain an answer in a standard form.

(c) What is the tangential acceleration?

Sought: a_T

$$a_T = \alpha_z r$$

$$a_T = (\underline{\quad\quad}) \cdot (2000mi)$$

$$a_T = 0$$

The spaceship's tangential acceleration (a_T) is the rate at which its tangential velocity (v) is changing.

Like angular velocity, angular acceleration (α) can be related to tangential acceleration by using the radius (r).

However, since the angular velocity is constant, there is no angular acceleration, and so no tangential acceleration

Fig. 3 Illustration of a worked example in the reading booklet

On the next page was a near transfer problem to solve (this problem was identical for all three conditions). This problem involved the same concepts as those used in the worked examples but was implemented with a different cover story (e.g., a jet ski). It is classified as a near transfer problem because it also requires the same equations and procedures as those demonstrated in the worked example. Furthermore, because the problem was given immediately after the worked example study and had the same problem format (e.g., wording and sequencing of question prompts) we expected students in all three conditions to see this as a continuation of what they were just learning about and to apply their knowledge from the previously studied worked examples. On the next page was the solution to this problem. This structure and organization was then repeated for the remaining two sets of worked examples. That is, the next pages consisted of two worked examples with instructional

A toddler wheels a toy truck around the outside of a circular sandbox of radius 1.15 m at a constant speed of 0.54 m/s. Assume the truck is moving in a counterclockwise direction, and that this is the positive direction.

a.) What is the angular velocity?
b.) What is the radial acceleration?
c.) What is the tangential acceleration?

Fig. 4 Illustration of a practice problem in the reading booklet

explanations followed by an isomorphic practice problem, followed by a near transfer problem that all three conditions received. This was then repeated for a third set of worked examples. In sum, students in the reading condition were given six worked examples, three practice problems, and three near transfer problems.

Self-explanation booklet The learning booklet for the self-explanation condition consisted of the same six worked examples without the fill-in-the-blanks used in the reading booklet. However, the instructional explanations were not initially given and students were instructed to "*Read aloud the problem and then explain aloud the reasoning or justification for each step of the solution*." They were told that they could use the principle booklets to help them generate their explanations and justifications for each step.

On the next page, the students were given the identical explanations and justifications that were given to the reading group without the fill-in-the-blanks. They were told to read the instructional explanations and try their best to understand them. On the next page, they were given the second worked example and were given the same instructions to generate explanations for each step of the given solution. The following page had the instructional explanations and justifications. Finally, they were given the near transfer problem to solve (the same problem as that given to the reading group). This material structure and design was then repeated for the remaining two sets of worked examples. In sum, participants were given six worked examples and three near transfer problems.

Analogy booklet The booklet for the analogy condition used the same six worked examples and instructional explanations as the reading booklet without the fill-in-the-blanks. Students were first instructed to read aloud the worked examples one at a time. The next page of the booklet consisted of a worksheet in which they were asked to compare and contrast the two examples writing out the similarities and differences between them.[3] There were four questions for the first compare and contrast activity. These questions were designed to focus the learner on important features of the underlying concepts. The questions typically proceeded from more general to more specific comparisons. For example, the set of questions for the first pair of examples began with the question: "*For Part A*, *what is similar and different across the problems*?" and ended with the question: "*From these problems*, *can*

[3] Note that this comparison and contrast activity comes after the initial introduction of principles and concepts and that the principles are available during the comparison (see Rittle-Johnson et al. 2009 for a similar approach). We view this as an important component of the instruction because it affords students the opportunity to use the principles to answer the comparison questions while working with the examples, i.e., providing the opportunity to constructively relate the principles to commonalities across examples. This is unlike some analogical comparison activities used in the literature that either do not give explanations or withhold explanations/principles until after the comparison task (e.g., Gick and Holyoak 1983). We will revisit this feature in the interpretation of the results in the "Discussion" section.

we say with certainty that a motion with constant speed is motion with no acceleration? *Explain your answer*". These prompts differ from the self-explanation prompts because they are designed to draw the learners' attention to the critical conceptual features that are invariant across the examples whereas the self-explanation prompts are focused on having students generating justifications for solution steps within a single problem.

The next two pages of the booklet contained the answers to the compare and contrast questions (see Fig. 5 for an example of answers to the first two questions). The students were instructed to read through the explanations and try their best to understand them. The next page presented the same near transfer problem as that given in the self-explanation and reading booklets. The following two sets of worked examples repeated this structure and

1.) For part A, what is similar and what is different across the two problems?

Both problems ask for the same sought quantity: ***angular velocity*** (W_z). The contexts across the two problems are similar (both involving vehicles navigating turns), but differ in that one is a spaceship and the other is a car. The angular velocity (W_z) is the rate at which the angular position (q_z) of the object is changing as it moves in a circle.

For both problems we choose the formula that relates angular velocity (W_z) with translational velocity (v), since we are given translational velocity of the object and radius of the circle (r) in the problem statements.

If we compare the resultant angular velocity (W_z) across problems (putting them in similar units), the angular velocity (W_z) of the spaceship is actually much smaller than the car. This is due to the very large radius (r) the spaceship is navigating relative to its translational velocity (v).

2.) For part B, are there differences in what the two problems ask for in terms of acceleration? If so, what are they?

There are no real differences between the two accelerations requested across problems. The spaceship problem asks for the components separately (radial followed by tangential acceleration), and the car problem asks for the magnitude and direction of the ***total acceleration*** (which is found by vector summation of the two components).

Radial acceleration (a_R) arises as a result of the constantly changing direction of the translational velocity (v). Since we already calculated the angular velocity (W_z) in part A of both problems, we take this value, square it, and multiply it by the radius (r) of the circle to find the radial acceleration (a_R).

The ***tangential acceleration*** (a_T) is the rate at which its tangential velocity is changing. Like angular velocity, angular acceleration (a_z) can be related to the tangential acceleration (a_T) by using the radius (r) of the circle. However, since the angular velocity (W_z) is **constant** there is no angular acceleration (a_z), and therefore no tangential acceleration (a_T). Thus the total acceleration is fully determined by the radial acceleration (a_R) since the tangential acceleration (a_T) is zero; this is true for both problems.

Fig. 5 Example explanations for the compare and contrast questions in the analogy booklet organization. In sum, they were given six worked examples, three comparison and contrast tasks, and three near transfer problems.

Test phase The test booklets consisted of two intermediate transfer problem solving tasks and a far transfer multiple choice test (13 questions). In accordance with the transfer taxonomy of Barnett and Ceci (2002), we classify the transfer distance of our tests based on the *problem content* in comparison to learning.[4] Barnett and Ceci classify procedural content as near transfer and conceptual and principle content as far transfer. Similarly, the memory demands are expected to increase from near to intermediate to far transfer.

Intermediate transfer: problem solving. The intermediate transfer problems assessed the identification and application of the relevant concepts and procedures from the worked examples for new problem contents (i.e., cover story). We classified these problems as intermediate transfer because they required the recognition of the conceptual features in the problem in order to execute and adapt the prior problem solving procedures (see Fig. 6). These problems should be more difficult than the near transfer problems for two reasons. First, the problems did not immediately follow the relevant worked examples so students had to determine which equations and procedures to apply for the given problem (i.e., they need to know the application conditions for a given equation). Second, the problems included new components that required students to either adapt their prior procedures or to ignore the additional irrelevant (and potentially misleading) information in the problem cover story.

For the first problem, students had to apply the concepts and equations from a previous worked example in a different way than they had practiced during the learning phase (a different set of steps). For students to be able to solve this problem correctly, they needed to identify the relevant equations and to be able to manipulate those equations to solve for a new variable. The second problem also required students to apply the same concepts and equations from a previous worked example. However, this problem also included some new irrelevant information (extraneous values) in the problem statement. Because this problem included more values than the prior problems from the learning phase, students had to know how to correctly map the values in the problem statement to the variables in the equations. This problem solving skill requires some conceptual knowledge for how quantities are instantiated in the equations.

Far transfer: multiple choice test. The far transfer test assessed qualitative reasoning and conceptual understanding in rotational kinematics (see Fig. 7 for two example problems). To solve the problems correctly, students needed to understand the relationships between concepts and to map them to the critical problem features. We consider this a far transfer assessment according to Barnett and Ceci (2002) because these problems required the recall, identification, and application of conceptual/principle knowledge from the worked examples. Furthermore, participants had not previously practiced solving such qualitative problems in their class or on homework assignments.

Procedure

The procedure consisted of a learning phase and a test phase. At the beginning of the learning phase, students were given an introductory principle booklet, calculator, digital

[4] A second factor in the taxonomy of Barnett and Ceci (2002) that can be used to define the transfer distance is the *problem context* at learning and test (e.g., temporal context, physical context, modality, etc.). All of our assessments fall in the near transfer category on this factor even though they vary in transfer distance on the factor of problem content.

Intermediate Transfer Problem – New Variable

A disc 6.45 cm in radius rotates at a constant rate of 99.0 rad/s about a central axis. Assume it rotates counterclockwise, and that this is the positive direction.

a.) What is the tangential speed of a point on the rim?
b.) Determine the total distance a point on the rim moves in 16.0s.
c.) Find the radial acceleration of a point 2.10 cm from the center.

Intermediate Transfer Problem – Irrelevant Features

A horse drawn carriage after traveling 24 miles at a constant speed of 7.81 m/s takes a circular turn of radius 85.5m around the palace at Versailles. Assume the circular motion is counterclockwise, and that this is positive direction.

a.) What is its angular speed?
b.) What is the magnitude of its tangential acceleration?
c.) What is the magnitude of its radial acceleration?

Fig. 6 Shows intermediate transfer problems 1 and 2

recorder, headset microphone, and pen. They were then given 9 min to read through the introductory booklet and had access to it through the entire learning phase. They were then given a practice think aloud task for 4 min that consisted of solving arithmetic problems while talking aloud. All three groups were instructed to talk aloud during the learning phase of the experiment. They were instructed to talk aloud to ensure that they were actually engaging in the activity (e.g., reading, self-explaining, comparing). Students were then randomly assigned to one of the three instructional conditions and received the corresponding learning booklet for that condition.

The timing and presentation of the learning activities is summarized in Fig. 1. All participants received the three pairs of worked examples in the same order. However, within each pair, the order of the examples was counterbalanced. Half the students received example 1 then example 2 and half received the opposite order.

Assume that you are observing a wheel that is rotating clockwise. What can you determine about the angular acceleration?

a.) It is positive
b.) It is negative
c.) Nothing unless you know the angular velocity
d.) Nothing unless you know how the rotation speed is changing

A car moving on a circular track increases its speed by a factor of two. Simultaneously, it doubles its radius of movement. Its radial acceleration will:

a.) Quadruple
b.) Double
c.) Halve
d.) Not change

Fig. 7 Two example problems from the multiple choice test

Students were given 12 min to work through the learning activities for the first two worked examples.[5] After which, they were given 5 min to solve the first near transfer problem and 2 min to study the corresponding solution. They were then given 6 min to work through the learning activities for worked examples 3 and 4, with 3 min to solve the second near transfer problem, and 2 min to study the corresponding solution. Participants were then given 15 min for examples 5 and 6, followed by 7 min to solve the third near transfer problem, and 3 min to study the solution. They were not permitted to refer to the example problems when solving the near transfer problems, although they did have access to the principle booklet. The total training time, including the introduction and example/practice activities, was 68 min. After the learning phase, all students were given the test to assess their understanding of the conceptual relations and their problem solving skills. They were given 14 min for the far transfer test and 5 min for each intermediate transfer problem. The test session was 24 min in length.

Results

The results are divided into two sections: learning and test. In the learning section, we examine the effect of the different instructional activities on student's performance on the near transfer problems. In the test section, we examine performance on the intermediate and far transfer tests. Alpha was set to 0.05 for all main effects, interactions, and planned comparisons (Keppel 1991). Effect sizes (partial eta squared, η_p^2) were calculated for main effects, interactions, and planned comparisons. Cohen (1988; see also Olejnik and Algina 2000) has suggested that effects be regarded as small when $\eta_p^2<0.06$, as medium when $0.06 <\eta_p^2<0.14$ and as large when $\eta_p^2>0.14$.

Learning: near transfer

In the learning phase, students in all three conditions solved the same near transfer problems presented after each set of worked example activities. These problems measure students' learning from the worked example activities because they were not allowed to page back or forward while solving each problem. However, they were allowed to use the principles booklet and their calculators.

For each problem, we assessed three different components of their solution including: the formula, variable instantiation, and the total score with final computation. The *formula score* measured whether they applied the appropriate formula to the problem and was scored as either incorrect or correct (0 or 1). This assesses whether they correctly classified the problem and used the correct equation. The *variable instantiation score* assessed whether students were able to correctly assign the appropriate values in the word problem to the variables in the equation. The scores varied from a two- to a four-variable maximum, one point was assigned for each correct instantiation. The *total score* averaged the formula score (0 or 1), variable instantiation score (proportion of variables correctly assigned), and the correct computation (0 or 1 for correctly computing the final answer and labeling with the correct units). These three measures were calculated for each part of every near transfer problem. All three measures revealed the same pattern of results so we report just the total score here.

[5] The specific amounts of time given for completing the various learning and test activities was determined by pilot testing the materials and procedures with a separate sample of students.

An initial analysis of each group's performance on the near transfer problems revealed that there was high within-group variability. Furthermore, analysis of the distribution showed that the sample violated normality assumptions (Shapiro–Wilk (N=55)=0.947, p< 0.05). The distribution for each condition showed strong right skews and inspection of the histograms showed that students at approximately the bottom third showed a marked decrease in performance. We used the upper two thirds specifically as the cutoff because analysis of the data revealed that the average difference between the lower third and upper two thirds was larger than other splits such as comparing the lower half versus upper half of students for all three conditions.

Therefore, to assess near transfer problem solving performance we used only those students who scored in the top two thirds within each group (reading, n=13; self-explanation, n=13; analogy, n=13). This effectively enforces a minimum performance criterion in order to test the success of the intervention for near transfer. Students in the upper two thirds had a *minimum score of 60 % percent correct* on the near transfer problems whereas those in the lower one third averaged 44 % correct. By restricting our analysis to the upper two thirds of performers, we have a manipulation check that participants were able to meet a minimum criterion level of success during learning. Furthermore, using the upper two thirds of the participants no longer violated normality assumptions of the distribution (Shapiro–Wilk (N=39)=0.961, *ns*). Importantly, we use these same students when we examine what is learned from the different forms of instruction in the test phase. Figure 8 shows the mean near transfer problem solving performance and standard errors for the upper two thirds of individuals in the three instructional conditions collapsed across problem type.

A three (problem type: 1, 2, 3) by three (instruction: reading, self-explanation, analogy) mixed analysis of variance (ANOVA) was conducted to examine the effect of instruction on students' total score as a function of problem type. There was a large effect of instruction, $F(2, 36)=4.41$, $p<0.05$, $\eta_p^2=0.19$. There was no effect of problem type, $F(2, 72)=1.48$, *ns*, or interaction of problem type by instruction, $F<1$. Follow-up planned comparisons for the effect of instruction showed that both the reading and self-explanation groups (M=0.84, SD=0.10 and M=0.85, SD=0.13, respectively) performed better than the analogy group (M=0.73, SD=0.10), $F(1, 25)=7.44$, $p<0.05$, $\eta_p^2=0.24$ and $F(1, 25)=6.47$, $p<0.05$, $\eta_p^2=$ 0.21, respectively.

These results are consistent with the hypothesis that reading worked examples with instructional explanations and solving practice problems facilitates learning the equations and procedures that can be successfully applied to solve similar problems. The fact that the

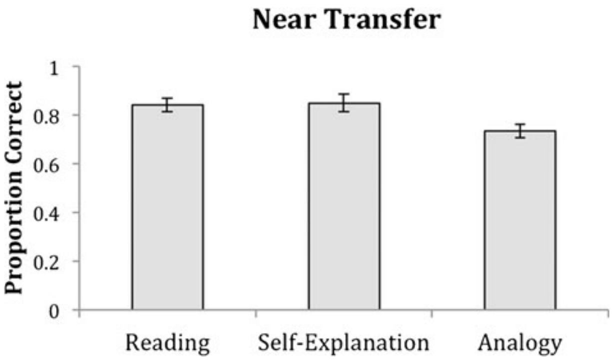

Fig. 8 Mean total score performance on the near transfer problems and standard errors for each group collapsed across problem type

self-explanation group also performs at a high level of accuracy is consistent with the interpretation that these individuals construct both a conceptual and a procedural, step-by-step, understanding of the problem solutions. The analogy group showed relatively poor performance suggesting that they were more focused on generating conceptual comparisons of the problem features across problems than learning the step-by-step set of procedures for a single problem. We hypothesized that these students would show relative improvements on the intermediate and far transfer tests when compared to the other groups.

Test: intermediate and far transfer

For both tests, we used the same upper two thirds of students from the learning phase. Using the upper two thirds of performers provides a strong test for determining what kind of knowledge was acquired from instruction when that instruction was successful. We already have some evidence that the reading and self-explanation instruction supports near transfer. Here, we ask whether the knowledge acquired during learning transfers to intermediate and far transfer problems.

Intermediate transfer The same three problem solving measures from the near transfer problems were used to assess performance on the intermediate transfer problems. We only report the total scores as the formula and variable instantiation scores showed the same pattern of results. Figure 9 shows the average performance of each group and standard errors on the two intermediate transfer problem solving tasks.

A two (problem type: new variable vs. irrelevant features) by three (instruction: reading, self-explanation, analogy) mixed ANOVA was conducted to examine the effect of instruction on problem solving performance as a function of problem type. There was no effect of problem type or instruction, F’s<1, and there was no interaction of instruction by problem type, $F(2, 36)=2.19$, $p=0.127$, $\eta_p{}^2=0.11$. These results show no significant differences across the conditions on the either of the intermediate transfer problems. This suggests that all three training conditions benefited equally from the worked example activities for intermediate transfer. Although the analogy group performed significantly worse than the reading and self-explanation groups on the near transfer problems this difference disappeared on the intermediate transfer problems. Next, we ask whether the conditions differed in their performance on far transfer test.

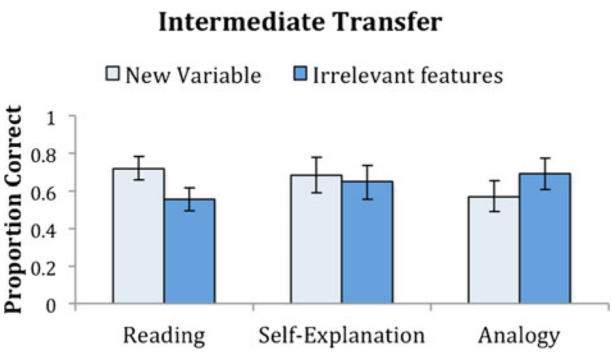

Fig. 9 Mean total score performance and standard errors for each group on the two intermediate transfer problems

Far transfer The multiple choice test assessed students' performance on problems that required qualitative reasoning. Each item was scored as either correct or incorrect. An analysis of the internal reliability of the test revealed a Cronbach's alpha of 0.59 (Cronbach 1951; Cortina 1993). An item analysis revealed that two items had poor reliability with very low corrected item-to-total correlations of less than 0.05. Dropping these two items from the analysis improved the reliability metric to alpha=0.63, achieving acceptable reliability (Schmitt 1996). To further evaluate the internal reliability of the measure, we analyzed the internal consistency of all items that had corrected-item-to-test correlations above 0.25 or higher, which left five items. This subset of items also had a Cronbach's alpha=0.63. Figure 10 reports the average performance and standard errors for each group on both the 11-item test and the five most reliable items.

For the 11-item test, we conducted a one-way between subjects ANOVA to examine the effect of instruction on test performance. The ANOVA revealed a large effect of instruction, $F(1,38)=3.31$, $p<0.05$, $\eta_p^2=0.16$. Follow-up planned comparisons showed that both the analogy and self-explanation groups ($M=0.56$, SD=0.22 and $M=0.59$, SD=0.24) performed better than the reading group ($M=0.41$, SD=0.13), $F(1, 25)=4.91$, $p<0.05$, $\eta_p^2=0.17$ and $F(1, 25)=6.44$, $p<0.05$, $\eta_p^2=0.21$, respectively. These results were also replicated for the five most internally reliable items, with a large effect of instruction $F(1,38)=5.54$, $p<0.05$, $\eta_p^2=0.24$. Follow-up planned comparisons again showed that both the analogy and self-explanation groups ($M=0.60$, SD=0.28 and $M=0.52$, SD=0.29) performed better than the reading group ($M=0.28$, SD=0.19), $F(1, 25)=11.60$, $p<0.05$, $\eta_p^2=0.33$ and $F(1, 25)=6.54$, $p<0.05$, $\eta_p^2=0.21$, respectively. There were no significant differences between the self-explanation and analogy groups.

These results are consistent with the hypothesis that generating explanations facilitated the construction of relations linking the abstract principles and concepts in the domain to the problem features. By generating explanations, participants acquired knowledge that enabled them to reason qualitatively about the concepts underlying the equations in the examples. Similarly, analogical comparison focused participants away from the surface features of the problem and toward the important qualitative relations that were common across the examples. Critically, this knowledge enabled them to engage in qualitative reasoning with tasks that had new problem features. Although the reading group showed high performance on the near and intermediate transfer problem solving tasks they showed comparatively worse performance on far transfer. This suggests that these participants developed knowledge of which kinds of equations go with particular problem types and could adapt these

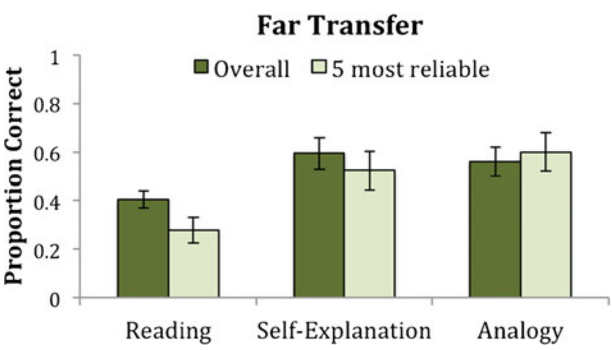

Fig. 10 Mean performance and standard errors for each group on the far transfer multiple-choice 11-item test and the five most reliable items

procedures but acquired less knowledge about how these features relate to the underlying principles and concepts.

Discussion

As predicted, the reading and self-explanation groups performed well on the near transfer problems. This result provides evidence that participants in the reading group acquired knowledge of the equations as well as the procedures for how to apply them to problems very similar to those they had already seen. This is consistent with prior research showing that including worked examples early in instruction is effective in facilitating later problem solving (Atkinson et al. 2003; Paas and Van Merrienboer 1994; Renkl and Atkinson 2003). These results also provide evidence that the students in the self-explanation group acquired a procedural understanding of the examples so that they could apply that knowledge to a very similar problem with high accuracy. In contrast, the analogy group performed less well (approximately 10 % worse) than the reading and self-explanation groups, showing that the knowledge they acquired was less helpful on near transfer problem solving. This result supports the hypothesis that the analogy group was more focused on the conceptual features across the problems than on the procedural steps within a problem. Further, if the analogy group acquired primarily declarative knowledge with little procedural knowledge then we would expect these participants to be more likely to make implementation or instantiation errors. Research on knowledge compilation has shown that although declarative knowledge is more flexible than procedural knowledge (i.e., can apply to multiple contexts), it also takes more time to apply to problem solving, and is more error prone (Anderson 1987; Anderson and Jeffries 1985; Nokes and Ohlsson 2005). To test the prediction regarding the flexibility of the knowledge acquired from learning, we turn to the performance on the intermediate and far transfer tests.

The difference observed on the near transfer problems was short lived as all three groups performed similarly on the intermediate transfer problems. The analogy group may have needed the problem solving practice during the learning phase to develop the procedural knowledge needed to support intermediate transfer, whereas the students in the other conditions acquired the procedural knowledge from example study. In contrast to our predications, the reading group performed similarly to the self-explanation and analogy groups on the intermediate transfer problems. One explanation for this finding is that the reading group acquired some amount of conceptual understanding for similar problem types that helped them reason about relevant versus irrelevant information and adapt prior procedures to solve for a new variable. This finding is consistent with the meta-analysis results showing that instructional explanations can support some conceptual learning (Wittwer and Renkl 2010).

On the far transfer test, the self-explanation and analogy groups performed better than the reading group. This provides evidence that students in both the self-explanation and analogy conditions acquired knowledge of the relations between principles and examples and were able to use that knowledge in applying those principles to problems with new features. In contrast, although the reading group was given the same examples with instructional explanations they did not perform as well on the far transfer test. This provides evidence that the reading group acquired knowledge that was more closely tied to the specific examples and problem features from the learning phase than the self-explanation and analogy groups.

In sum, these findings show that when students are learning from worked examples they can benefit from instruction that helps them connect those examples with the underlying principles and concepts. This may be especially important in domains like physics where students often have misconceptions of the target concepts and principles before receiving instruction (e.g., McClosky and Kohl 1983). In contrast, for domains in which students have a good prior intuition of the target concepts they may show benefits from receiving both examples and rules without additional assistance to help link the two (Fong and Nisbett 1991; Fong et al. 1986). It is interesting to consider that some types of prior knowledge and experience may better prepare students to spontaneously engage in constructive cognitive processes linking principles and examples when receiving new instruction. However, in the absence of such knowledge, the current results show that instructional interventions can help scaffold those processes. Future work should further examine the interaction between prior knowledge, the amount of instructional assistance, and transfer (see Koedinger and Aleven 2007, for a recent review of this issue for intelligent tutoring systems).

This work shows that even a very brief instructional intervention can lead to improvements in conceptual learning in a classroom setting above and beyond the typical classroom instruction (i.e., reading principles, studying examples with instructional explanations, and solving practice problems). In addition, this work shows that cognitive science principles discovered in the laboratory such as self-explanation and analogical comparison can create measurable differences in classroom learning. We view each of the instructional interventions as having benefits for learning, albeit of different kinds.

Each instructional pathway creates a different kind of knowledge and skill. Although the reading group performed poorly on far transfer problems they performed quite well on the near and intermediate transfer problems. This suggests they acquired a good procedural understanding of the worked examples as well as some ability to adapt that knowledge to solve new problem solving tasks. In contrast, the self-explanation group performed well on all three types of transfer problems, suggesting that these students developed both procedural and conceptual knowledge from the learning activities. This provides evidence that these students were able to connect the problem features and steps of the examples to the underlying physics concepts and ideas expressed in the written principles through self-explanation. Finally, the analogical comparison group showed initial poor performance on near transfer problems, but better performance on the intermediate and far transfer problems. These results show that analogy students had initially little procedural knowledge but had acquired a conceptual understanding of the examples that helped them on the qualitative reasoning problems and with practice supported problem solving transfer. Importantly, to be able to make these observations, we needed fine-grained assessments to differentiate what is learned using near, intermediate, and far transfer tasks (for similar assessments see Belenky and Nokes 2009; Mestre et al. 2009).

In contrast to our predictions, the results did not provide any support for the hypothesis that the analogy group would acquire more abstract conceptual knowledge than the self-explanation group. If they had acquired more abstract knowledge, we would have expected them to perform better than the self-explanation group on the intermediate and far transfer problems. It is possible that our measures did not effectively assess the abstract nature of their knowledge. Perhaps measurements of long-term retention or future learning would reveal such differences, as more abstract knowledge should transfer to new contexts and help in learning new related concepts (e.g., Belenky and Nokes-Malach 2012; Bransford and Schwartz 1998). Another possibility is that both groups acquired equally abstract knowledge. It may be the case that our particular instantiation of case comparisons was not optimal for acquiring abstract knowledge. As mentioned in the "Methods" section, we provided students the principles both before and simultaneously with the case comparisons. It would be interesting to see whether there would be further benefits for abstraction if the principles were provided after the case comparisons. Future work should further explore the similarities and differences between what is learned from these different constructive activities and the implications for problem solving and instruction. Once we know the tradeoffs for different instructional tasks in terms of what is learned, we can design curricula with the right proportion of time spent on different tasks to achieve the learning goals of the teacher, school, district, and state.

In vivo experiments are critical to bridging research in the cognitive and learning sciences to education. Not only do they help us test hypotheses relating cognitive outcomes to instruction but they also facilitate asking new questions and raising new issues that can then be brought back into the laboratory and investigated. For example, individual differences are a perennial issue of concern in learning and instruction. This becomes especially salient in a classroom setting when many interacting variables arise such as prior knowledge, motivation, and cognitive ability. This was also the case in the current work in which we focused on the effect of instruction for the upper two thirds of learners. How do these different types of instruction interact with motivation, prior knowledge, or cognitive ability? Future work should further examine these issues in both in vivo classroom and in laboratory settings in order to build a cumulative science that will have an impact on classroom instruction.

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Current themes of research:

Conceptual learning. Problem-solving. Knowledge transfer. Dialectical interaction. Competence motivation. Educational technology.

Most relevant publications in the field of Psychology of Education:

Gadgil, S., & Nokes-Malach, T. J. (2012). Collaborative facilitation through error-detection: A classroom experiment. *Applied Cognitive Psychology, 26*(3), 410–420.
Gadgil, S., Nokes-Malach, T. J., & Chi, M. T. H. (2012). Effectiveness of holistic mental model confrontation in driving conceptual change. *Learning and Instruction*, *22*, 47–61.
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Current themes of research:

Intelligent tutoring systems. Computational cognitive modeling. Applications of artificial intelligence to education. Educational data mining. Cognitive science. Learning sciences.

Most relevant publications in the field of Psychology of Education:

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Current themes of research:

Role of interest and motivation in learning. Abstract/concrete learning materials. Transfer. Mental set. Metacognition.

Most relevant publications in the field of Psychology of Education:

Belenky, D. M., & Nokes-Malach, T. J. (2012). Motivation and transfer: The role of mastery-approach goals in preparation for future learning. *Journal of the Learning Sciences, 21*(3), 399–432.

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Current themes of research:

Cognitive science. Conceptual learning. Physics education.

Most relevant publications in the field of Psychology of Education:

NA.

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Current themes of research:

Memory, learning, and the nature of mental representations.

Most relevant publications in the field of Psychology of Education:

Cox, G. E., & Shiffrin, R. M. (2012). Criterion setting and the dynamics of recognition memory. *Topics in Cognitive Science, 4*(1), 135–150.